K.S.R COLLEGE OF ENGINEERING (AUTONOMOUS), TIRUCHENGODE -637215

18MA344 -Differential Equations and Numerical Methods

TWO MARKS QUESTION BANK

UNIT – I

PARTIAL DIFFERENTIAL EQUATIONS

1. Form the PDE by eliminating 'a' and 'b' from $z = (x + a)(y + b)$.
   By differentiating the given, with respect to 'x' and 'y' , we get, $p = y + b$ and $q = x + a$.
   By substituting this in the given equation, we get, $z = pq$, which is the required PDE.
2. Form the PDE by eliminating 'a' and 'b' from $z = ax + by + a^2 + b^2$.
   By differentiating the given, with respect to 'x' and 'y' , we get, $p = a$ and $q = b$.
   By substituting this in the given equation, we get, $z = px + qy + p^2 + q^2$, which is the required PDE
3. Form the PDE by eliminating 'a' and 'b' from $z = a(x + y) + b$.
   By differentiating the given, with respect to 'x' and 'y' , we get, $p = a$ and $q = a$.
   By substituting this in the given equation, we get, $p = q$, which is the required PDE
4. Form the PDE by eliminating 'f' from $z = f(x^2 + y^2)$.
   By differentiating the given, with respect to 'x' and 'y' , we get,
   $p = f'(x^2 + y^2).2x \quad \text{and} \quad q = f'(x^2 + y^2).2y$
   Dividing these two, we get, $py = qx$, which is the required PDE
5. Form the PDE by eliminating 'f' from $z = f(xy)$.
   By differentiating the given, with respect to 'x' and 'y' , we get,
   $$p = f'(xy).y \quad \text{and} \quad q = f'(xy).x$$
   Dividing these two, we get, $px = qy$, which is the required PDE
6. Form the PDE by eliminating arbitrary functions from $z = f(x) + g(y)$.
   By differentiating the given, with respect to 'x' and 'y' , we get,
   $$p = f'(x), q = g'(y), r = f''(x), s = 0, t = g''(y).$$
   From this, $s = 0$, which is the required PDE
7. Form the PDE by eliminating arbitrary functions from $z = y\,f(x) + g(x)$.
   By differentiating the given, with respect to 'x' and 'y' , we get,
   $$p = yf'(x) + g'(x), \quad q = f(x), \quad r = yf''(x) + g''(x), \quad s = f'(x), \quad t = 0.$$
   From this, $t = 0$, which is the required PDE

8. The complete solution of the PDE $pq = k$ is ………………. Answer: $z = ax + \frac{k}{a}y + b$

9. The complete solution of the PDE $p = e^q$ is ………………. Answer: $z = ax + y\log a + b$
10. Solve $\frac{\partial^2 z}{\partial x^2} = xy$.
    By direct integration, we get, $z = \frac{x^3 y}{6} + f(y) + xg(y)$ where f(y) and g(y) are arbitrary functions.
11. Solve $\frac{\partial^2 z}{\partial y^2} = \cos(2x + 3y)$.
    $z = y\,f(x) + g(x) - \frac{\cos(2x+3y)}{9}$
12. Solve $\left(9D^2 + 6DD' + D'^2\right)z = 0$.
    CF = $x\,f_1\left(y - \frac{x}{3}\right) + f_2\left(y - \frac{x}{3}\right)$, PI = 0, So $z = x\,f_1\left(y - \frac{x}{3}\right) + f_2\left(y - \frac{x}{3}\right)$ which is the required solution.
13. Obtain PDE by eliminating arbitrary constants 'a and b' from $(x-a)^2 + (y-b)^2 + z^2 = 1$.
    Partially differentiating the given equation with respect to 'x ' and 'y', we get,
    $$x - a = -z\,p \text{ and } y - b = -z\,q$$
    Substituting x-a and y-b in the given equation, we get,
    $p^2 + q^2 + 1 = \frac{1}{z^2}$ This is the required PDE.
14. Form the PDE of all spheres whose centers lie on the z-axis.
    The Equation of such sphere is $x^2 + y^2 + (z - c)^2 = r^2$ … … (I) , where r & c are constants
    Partially differentiating I with respect to 'x ' and 'y', we get,

$2x + 2(z-c)p = 0$ .... (II)

$2y + 2(z-c)q = 0$ ie., $(z-c) = -\frac{y}{q}$ .... (III)

Substituting (III) in(II) we get, xq=yp which is the required PDE

15. Find the complete integral of the PDE $(1-x)p + (2-y)q = 3-z$.

This is of the type $z = px + qy + f(p,q)$

The given eqn can be written as

$Z = px + qy - p - 2q + 3$

The complete integral is

$Z = ax + by - a - 2b + 3$ I

To find singular integral :

Partially differentiating I with respect to 'a ' and 'b' and then equating to zero we get,

$\frac{\partial z}{\partial a} = x - 1 = 0$ ie., $x = 1$ II

$\frac{\partial z}{\partial b} = y - 2 = 0$ ie., $y = 2$ III

To find singular integral we have to eliminate 'a' & 'b' from I, II & III. For substituting II & III in I we get z = 3, which gives the singular integral.

16. Form the PDE by eliminating arbitrary function from $\phi(z^2 - xy, x/z) = 0$.

Let $u = z^2 - xy$ $V = \frac{x}{z}$

Then the gn eqn is of the form $\phi(u,v) = 0$

The elimination of ϕ from the above eqn we get $\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$

$\begin{vmatrix} 2zp - y & \frac{z-px}{z^2} \\ 2zq - x & \frac{-xq}{z^2} \end{vmatrix} = 0$ ie., $px^2 - q(xy - 2z^2) = zx$

17. Find the general solution of $4(\partial^2 z/\partial x^2) - 12(\partial^2 z/\partial x\partial y) + 9(\partial^2 z/\partial y^2) = 0$.

A.E is $4m^2 - 12m + 9 = 0$ (replacing D by m and D' by 1)

$m = \frac{3}{2}, \frac{3}{2}.$ $\therefore$ General solution is $z = f_1(y - \frac{3}{2}x) + x f_2(y - \frac{3}{2}x)$

18. Find PI of $\left(D^2 - 3DD' + 2D'^2\right) z = \mathrm{Cos}(x + 2y)$.

PI = $\frac{-\cos(x+2y)}{3}$

UNIT –II
SOLUTION OF EQUATIONS

1. If g(x) is continuous in [a , b],then under what condition the iterative method x = g (x) has a unique solution in [a, b]? (Remembering)

   $|g'(x)| < 1$ in [a, b]

2. Compare Gauss elimination and Gauss –Seidel methods for solving linear systems of the form AX = B.

   Gauss elimination method is direct method. Gauss –Seidel method is iterative method.

3. State the condition for the convergence of Gauss –Seidel method.(Understanding)

   Coefficient matrix should be diagonally dominant.

4. Find an iterative formula to find $\sqrt{N}$,where N is a positive number.(Remembering)

   $$x_{n+1} = \frac{x_n^{\ 2} + N}{2x_n}.$$

5. State the order of convergence and convergence condition for Newton's Raphson method.(R,co1)

   Order of convergence is two. Convergence condition is $|f(x) f''(x)| < |f'(x)^2|$.

6. Write the iterative formula for Newton's Raphson method.(R,CO1)

$$x_{n+1} = x_n - \frac{f(x_n)}{f^{'}(x_n)}$$, n = 0,1,2,...

7. Write the condition for the convergence of iterative method.(R,CO1)
   $|g^{'}(x)| < 1$
8. Newton's Raphson method is otherwise known as -----------.(Method of tangents)
9. State the condition for the convergence of Gauss seidel method of iteration ,for solving the system of simultaneous equations.(U,co1)
   Coefficient matrix should be diagonally dominant.
10. In Newton Raphson method the error at any stage is proportional to the _______ of the error in the previous stage. (Square)
11. Given $y^1 = y - x^2, y(0) = 1, \ find \ \ y(0.1)$ by Taylor's method.(E,CO4)
    Ans: 1.10482
12. Write the algorithm for Euler's method. (R,CO4)
    Ans: $y_{n+1} = y_n + hf(x_n, y_n)$
13. What are limitations of Euler's method? (U,CO4)
    Ans: 1.The attainable accuracy is limited by length of step h.
    2.The method is slow and limited accuracy.
14. Given $y^1 = y - x^2, y(0) = 1, \ find \ \ y(0.2)$ by Modified Euler's method. (E,CO4)
    Ans: 1.218
15. Write the first order Runge Kutta method. (R,CO4)
    Ans: $y_1 = y_0 + h(x_0.y_0), \quad y_1 = y_0 + k_1$
16. Write the fourth order Runge Kutta method. (R,CO4)
    Ans: $k_1 = hf(x_n, y_n) \quad , \quad k_2 = hf(x_n + h/2, y_n + k_1/2)$
    $k_3 = hf(x_n + h/2, y_n + k_2/2), \quad k_4 = h(x_n + h, y_n + k_3)$
    $$\Delta y_n = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad y_{n+1} = y_n + \Delta y_n, \qquad n = 0,1,2,3,.......$$
17. Given $y^1 = 1 - y, y(0) = 0$ find $y(0.1)$ by R.K method. (E,CO4)
    Ans: y(0.1)=0.1
18. What do you meant by single step methods Give Example. (U,CO4)
    Ans: Taylor's, Euler's, Modified Euler's, Runge Kutta …. etc.

UNIT – III
INTERPOLATION AND APPROXIMATION

1. Define Interpolation and Extrapolation.(Remembering)
   The process of finding the value of a function inside the given range is called Interpolation.
   The process of finding the value of a function outside the given range is called Extrapolation.
2. Why the polynomial interpolation is preferred mostly?(Remembering)
   a. They are simple forms of functions which can be easily manipulated.
   b. Computations for definite values of the argument, integration and differentiation of such Functions are easy.
   c. Polynomials are free from singularities where as rational functions or other types, do have Singularities.
3. Write the Newton's forward interpolation formula. (Remembering)

$$Y(x) = y_o + \frac{u}{1!}\Delta y_o + \frac{u(u-1)}{2!}\Delta y_o + \frac{u(u-1)(u-2)}{3!}\Delta^2 y_o + ... + \frac{u(u-1)...(u-(n-1))}{n!}\Delta^n y_o .$$

Where $u = \frac{x - x_o}{h}$.

In the above formula, the first two terms will give the linear interpolation and the first three terms will give a parabolic interpolation.

4. Write Newton's Backward Interpolation formula. (Remembering)

$$Y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^2 y_n + \ldots + \frac{v(v+1)\ldots(v+(n-1))}{n!}\nabla^n y_n .$$

Where $v = \frac{x - x_n}{h}$.

5. State Newton's divided difference formula.(Understanding)

$f(x)= f(x_o) + (x-x_o)f(x_o,x_1) + (x-x_o)(x-x_1)\, f(x_o,x_1,x_2) + \ldots + (x-x_o)(x-x_1)\ldots(x-x_{n-1})\, f(x_o,x_1,x_2\ldots x_n)$.

6. State the error in Newton's forward interpolation and Newton's backward interpolation formula. (Understanding)

Error in forward interpolation =

$$f(x) - p_n(x) = \frac{u(u-1)(u-2)\ldots(u-n)}{(n+1)!}h^{n+1}f^{n+1}(c), where u = \frac{x-x_o}{h} .$$

Error in backward interpolation =

$$f(x) - p_n(x) = \frac{v(v+1)(v+2)\ldots(v+n)}{(n+1)!}h^{n+1}y^{n+1}(c), where \quad v = \frac{x-x_n}{h} .$$

7. Find the quadratic polynomial that fits $y(x) = x^4$ at x = 0,1,2. (Remembering)
8. If $u_1=1$, $u_3=17$, $u_4=43$, $u_5=89$ then find u_2. (Remembering)
9. Evaluate y(1) from (Evaluating)

X :	0	2	3
Y:	-1	3	5

10. Find the difference table for(Remembering)

X :	3	5	7	9
Y:	6	24	58	108

11. What is the Lagrange's formula to find y if three sets of values (x_o,y_o),(x_1,y_1) and (x_2,y_2) are given?.(Understanding)
12. State Newton's forward and backward interpolation formula. (Understanding)
13. State Newton's divided difference interpolation formula. (Understanding)
14. If f(x) = 1/x, find the divided differences f(a,b) and f(a,b,c).(Evaluating)
15. State inverse Lagrange's Interpolation formula. (Understanding)

X=

$$\frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_o-y_1)(y_o-y_2)\ldots(y_o-y_n)}x_o + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}x_1 + \ldots + \frac{(y-y_o)(y-y_1)\ldots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\ldots(y_n-y_{n-1})}x_n$$

16. When will you use Newton's backward interpolation formula? (Understanding)

We can apply the Newton's backward interpolation if the unknown value lies near the end of the table value.

17. Using Lagrange's interpolation formula, find the polynomial for (Applying)

x	0	1	3	4
y	-12	0	0	12

y = f(x) =

$$\frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_o-x_1)(x_o-x_2)\ldots(x_o-x_n)}y_o + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1 + \ldots + \frac{(x-x_o)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

(i.e) y =

$$\frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)\ldots(0-4)}(-12) + \frac{(x)(x-3)(x-4)}{(1-0)(1-2)(-4)}0 + \frac{x(x-1)(x-4)}{(3-0)(3-1)(3-4)}0 + \frac{x(x-1)(x-3)}{(4-0)(4-1)(4-3)}x$$

(i.e.) $y(x) = 2x^3 - 12x^2 + 22x - 12$.

18. Obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula.(Applying)

x	0	2	4	6
y	-3	5	21	45

Solution : The finite difference table is

X	y	Δ y	Δ^2y	Δ^3y
0	-3			
2	5	8		
4	21	16	8	
6	45	24	8	0

u = $\frac{x-0}{2}=\frac{x}{2}$. Therefore f(x) = y_o + u Δy_o + $\frac{u(u-1)}{2!}\Delta^2 y_o = x^2+2x-3.$

19. Find the parabola of the form y = ax^2+bx+c passing through the points (0,0) (1,1) and (2,20).(Evaluating)

By Lagrange's formula y = $\frac{(x-1)(x-2)}{(0-1)(0-2)}(0)+\frac{(x-0)(x-2)}{(1-0)(1-2)}1+\frac{(x-0)(x-1)}{(2-0)(2-1)}20$ = $9x^2 - 8x$.

20. State any two properties of divided difference. (Understanding)
   (i) The divided differences are symmetrical in all their arguments.
   (ii) The operator Δ is linear.
   (iii) nth divided differences of a polynomial of degree 'n' are constants.
21. What are the natural or free conditions in Cubic Spline.?(Understanding)
   Soln.: $s''(x_o) = 0$, $s''(x_n) = 0$ are called free conditions.
22. State the properties of Cubic Spline. (Understanding)
   (i) $S(x_i) = y_i$, i = 0 , 1 , 2 …. n
   (ii) S(x) , S'(x) , S"(x) are continuous in [a,b].
   (iii) S(x) is a cubic polynomial.
23. State the order of convergence of Cubic Spline. (Understanding)

   Order of convergence = 4. (Fourth order of convergence).
24. Find the divided difference of f(x) = $x^3 - 2x$ for the arguments 2,4,9,10.(Remembering)
25. If f(x) = $\frac{1}{x^2}$,then find f(a,b) .(Remembering)
26. Construct the divided difference table for the following.(Creating)

X :	1	2	4	7
Y:	22	30	82	106

27. A third degree polynomial passes through (0,-1) (1,1) (2,1) and (3,-2) then its value at x = 4 is
28. Show that the divided differences are symmetrical in their arguments.(evaluating)

   $f(x_o,x_1) = \frac{f(x_1)-f(x_o)}{x_1-x_o}=\frac{f(x_o)-f(x_1)}{x_o-x_1}=f(x_1,x_o).$

   Similarly we can prove that $f(x_1,x_2) = f(x_2,x_1)$.
29. Find the divided difference table for the following data.(Remebering)

X:	5	7	11	13	17
Y:	150	392	1452	2366	5202

30. Define forward, backward and divided difference.(Remebering)

   $\Delta f(x) = f(x+h) - f(x)$ $\qquad \nabla f(x) = f(x) - f(x-h)$

   $\Delta f(x) = \frac{f(x_1)-f(x_o)}{x_1-x_o}.$

UNIT IV
NUMERICAL DIFFERENTIATIATION AND INTEGRATION

1. What are the eerors in Trapezoidal and simpson's rules of numerical integration?(U,co3)

   The error in the Trapezoidal is $E < \frac{(b-a)h^2}{12} y''(\xi)$

   The error in Simpson's rule is $E < \frac{-h^4}{180}(b-a)$

2. Using Newton's backward difference, write the formulae for the first and second order derivatives at x = x_n (U,co3)

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left(\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \ldots\ldots\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n} = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \ldots\ldots\right)$$

3. In order to evaluate by Simpson's $\frac{1}{3}$ rule as well as by Simpson's $\frac{3}{8}$ rule,what is the restriction on the number of intervals? (U,co3)

   In Simpson's $\frac{1}{3}$ rule the number of sub intervals should be even.In Simpson's $\frac{3}{8}$ rule the number of sub intervals should be a multiple of 3.

4. Write the Simpson's th formula. (R,co3)

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8}\{(y_0 + y_n) + 3(y_1 + y_2 + \ldots\ldots\ldots + y_{n-1}) + 2(y_3 + y_6 + \ldots\ldots\ldots + y_{n-3})\}.$$

5. Write down the expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = x_0 by Newton's forward difference formula. (R,co3)

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left(\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \ldots\ldots\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left(\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \ldots\ldots\right)$$

6. Evaluate $\int_1^4 f(x)dx$ from the table by Simpson's $\frac{3}{8}$ rule. (E,co3)

x	1	2	3	4
f(x)	1	8	27	64

   By Simpson's $\frac{3}{8}$ rule, $\int_1^4 f(x)dx = \frac{3}{8}((1+64)+3(8+27))$

7. Using Trapezoidal rule evaluate $\int_0^{\pi} \sin x dx$ by dividing the range into 6 equal parts. (E,co3)

   Ans : 1.97

8. State Trapezoidal rule. (R,co3)

$$\int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2}((y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots + y_{n-1})).$$

9. Error in Trapezoidal rule is of order --------. (h^2) (U,co3)
10. Error in Simpson's rule is of order-----------------. (h^4) (U,co3)
11. A curve passes through (0,1),(0.25,0.9412),(0.5 , 0.8),(0.75,0.64) and (1,0.5).Find the area between the curve x- axis and x = 0 and 1 by trapezoidal rule. (U,co3)

    Ans :0.7828

12. Which one is more reliable Simpson's rule or Trapezoidal rule? (U,co3)
Simpson's rule.

UNIT V
BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

1. Express $xy'' + y = 0$ in terms of finite difference coefficients. (U,CO5)
   Ans: $x_i(y_{i+1} - 2y_i + y_{i-1}) + h^2 y_i = 0$

2. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation $u_{tt} = a^2 u_{xx}$. (U,CO5)
   Ans: For $\lambda = 1/a$, the solution of the difference equation is stable and coincides with the solution of the differential equation. For $\lambda < 1/a$, the solution is stable but not convergent.

3. Fill up the blanks:
   a) Explicit method is stable only if λ .... ($\lambda < 1/2$)
   b) Implicit method is convergent when λ .... ($\lambda = 1/2$)

4. State the conditions for the equation; $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$ where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic. (U,CO5)
   Ans: The given equation is said to be

   (i) Elliptic at a point (x,y) in the plane if $B^2 - 4AC < 0$
   (ii) Parabolic if $B^2 - 4AC = 0$
   (iii) Hyperbolic if $B^2 - 4AC > 0$.

5. What is the classification of $f_x - f_{yy} = 0$? (U,CO5)
   Ans: Here A = 0, B = 0, C = -1 $\Rightarrow$ $B^2 - 4AC = 0$

   So the equation is Parabolic.

6. What is the truncation error of the central difference approximation of $y'(x)$?
   Ans: The round-off error is the quantity R(say) which must be added to a finite representation of a computed number in order to make it the exact representation of that number.
   Y(computed number) + R = y(true number). The truncation error is the quantity T which must be added to the true representation of the computed quantity in order that the result must be exactly equal to the quantity we are seeking to generate Y(true number) + T= y(exact number).

7. For what value of λ, the explicit method of solving the hyperbolic equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t^2}$ is stable, where $\lambda = \frac{c\Delta t}{\Delta x}$? Ans: when $\frac{c\Delta t}{\Delta x} > 1$. (R,CO5)

8. What is the error for solving laplace and poisson's equations by finite difference method? (U,CO5)
   Ans: The error in replacing $\frac{\partial^2 u}{\partial x^2}$ by the difference expression of the order $o(h^2)$. Since h = k, the error in replacing $\frac{\partial^2 u}{\partial y^2}$ by the difference expression is of the order o(h2). Hence the error for solving Laplace and Poisson equation is o(h2).

9. Write down the Bender – Schmidt recurrence relation for one dimensional heat equation. (R,CO5)
   Ans: The Bender – Schmidt recurrence relation for one dimensional heat equation is
   $$u_{i,j+1} = \frac{1}{2}\left[u_{i+1,j} + u_{i-1,j}\right].$$

10. Write down the Crank – Nicholson formula to solve $u_t = u_{xx}$. (R,CO5)

Ans: The Crank – Nicholson formula is $u_{i+1,j+1} - 4u_{i,j+1} + u_{i-1,j+1} = u_{i+1,j} - u_{i-1,j}$.

11. Write the diagonal five – point formula to solve the Laplace equation $u_{xx} + u_{yy} = 0$. (R,CO5)

Ans: The diagonal five – point formula is $u_{i,j} = \frac{1}{4}\left[u_{i,j} + u_{i+i,j} + u_{i,j-1} + u_{i-1,j}\right]$.

12. Obtain the finite difference scheme for the difference equation $2\frac{d^2y}{dx^2} + y = 5$.(U,CO5)

Ans: The given differential equation can be written as $2\frac{d^2y}{dx^2} + y = 5$

Using the central difference approximation, we have $2y'' = \frac{y_{k-1} - 2y_k + y_{k+1}}{h^2}$

Substitution we get, $\frac{y_{k-1} - 2y_k + y_{k+1}}{h^2} + y_k = 5$

$\Rightarrow \quad y_{k-1} - 2y_k + y_{k+1} + y_k h^2 = 5h^2$

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