QUESTION BANK
MA 2261 - PROBABILITY AND RANDOM PROCESSES

UNIT – I –RANDOM VARIABLES

2 MARKS

1. Define Random variable.
   A random variable is a function that assigns a real number to each outcome in the sample space for random experiment.
2. Define discrete random variable with an example.
   A random variable whose set of possible values is either finite or countably infinite is called the Discrete.
3. What is the Probability mass function?
   For a DRV 'X' with possible values $x_1, x_2, x_3, \dots . x_n$, ***a probability mass function is a function such that***
   a) $P(X = x_i) \geq 0$ ***b)*** $\sum_{i=1}^{n} P(X = x_i) = 1$
4. Define Cumulative distribution function of a DRV.
   The CDF of a DRV is denoted as F(x) such that F(x) = P(X ≤ x)
5. What do u mean by the Mean of a DRV?
   The mean or an expected value of a DRV, is denoted as $\mu = E(x) = \sum_x x\, P(x)$
6. Probability density function f(x) can be used to describe the probability of a --------- random variable X.
   Answer : Continuous
7. Write short notes on Probability density function.
   For a CRV 'X', a pdf is a function such that
   a) $f(x) \geq 0$ ***b)*** $\int_{-\infty}^{\infty} f(x) = 1$
8. The Cumulative distribution function of a CRV can be defined as --------------------
   Answer: $F(x) = \int_{-\infty}^{x} f(x)dx$
9. Define expected value of a CRV.
   Answer : The mean or an expected value of a CRV, is denoted as $\mu = E(x) = \int_{-\infty}^{\infty} xf(x)dx$
10. The Pdf of a random variable X is f(x) = 2x, 0 < x <1, find the p.d.f of Y = 3x + 1
    Answer: Given Y = 3x + 1, $\frac{y-1}{3}$ **= x,** $\frac{dx}{dy} = \frac{1}{x} = \frac{2}{9}(y-1), 1 < y < 4.$
11. If a random variable X takes the value 1,2,3,4 suchthat 2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4).Find the probability distribution of X.
    Answer: Let P(X = 3) = k,then P(X = 1) = $\frac{k}{2}$, P(X = 2) = $\frac{k}{3}$, P(X = 4) = $\frac{k}{5}$
    ∴ The total probability is 1,we have P(X = 1) + P(X = 2)+ P(X = 3)+ P(X = 4) = 1
    $\therefore k = \frac{30}{61}$
12. A random variable is a --------- function. ***Answer: Single valued***
13. A random variable is not a ------------ function. ***Answer : Multi valued***
14. The probability mass function cannot take -------------- values. ***Answer: Negative***
15. For a DRV, the probability density function represents -------------. ***Answer: Probability mass function.***
16. The probability distribution function cannot have ------------- values. ***Answer: Negative***
17. The relationship between probability distribution function F(x) and the probability density function f(x) is -----------------. $f(x) = \frac{d}{dx}F(x)$
18. $E(aX + b) =$ -------------------- ***Answer***: $aE(x) + b$
19. $E(X - \bar{X}) =$ ------ ***Answer: Zero***
20. The other name of the moments about origins is --------------- ***Answer: Raw moments***
21. The other name of the moments about mean is ---------------- ***Answer: Central moments***
22. Define Moment generating function.

$$M(t) = E(e^{tx}) = \begin{cases} \sum_x e^{tx} P(x) \, if \ X \ is \ a \ DRV \\ \int_{-\infty}^{\infty} e^{tx} f(x) dx \ \ if \ X \ is \ a \ CRV \end{cases}$$

23. If the random variable X is uniformly distributed over (-1,1),Find the density function of y = sin$\left(\frac{\pi x}{2}\right)$
    Answer: since X is uniformly distributed over (-1,1) its p.d.f is f(x) = $\frac{1}{2}, -1 < x < 1$
    The p.d.f is given ***by f(y) =*** $\frac{d}{dy}[F(y)] = \frac{1}{\pi\sqrt{1-y^2}}, -1 < y < 1$

24. Binomial distribution is ------------------- if p = q = ½. ***Answer: symmetrical***.
25. For, binomial distribution is variance --------- mean. ***Answer: Less than***
26. Binomial distribution is ------------. ***Answer: Not continuous***
27. If X is a poisson variate suchthat P(X = 2) = 9P(X = 4) +90 P(X = 6),Find the variance

Answer: We know that P(X = x) = $\frac{e^{-\lambda}\lambda^x}{x!}$, x = 0,1,2…………n and $\lambda > 0$

$$\frac{e^{-\lambda}\lambda^2}{2!} = 9\frac{e^{-\lambda}\lambda^4}{4!} + 90\frac{e^{-\lambda}\lambda^6}{6!} \Longrightarrow \boldsymbol{\lambda = 1}$$

28. Geometric distribution has -----------. ***Answer: No memory***
29. Mean, Variance and third central moment of Poisson distribution are ---------- ***Answer: equal***
30. Poisson distribution is not a ----------- distribution. ***Answer: Symmetrical***
31. A binomial random variable is approximated to Poisson random variable when sample value is -------- and probability is close -----------. ***Answer: large, zero***.
32. Exponential distribution is a special case of -----------------. ***Answer: Gamma distribution***
33. For a normal distribution, coefficient of skewness is ------------. ***Answer: zero.***
34. For a binomial distribution, mean is 6 and standard deviation is $\sqrt{2}$, find 'n'. ***Answer: 9***
35. If 'X' is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90\,P(X = 6)$, the variance is -----.
    Answer: 1
36. A CRV 'X' has pdf given by $f(x) = 3x^2, 0 \le x \le 1$.Find K such that $P(x > K) = 0.05$.
    Answer: 0.9830
37. The graph of the normal distribution is ------------------. ***Answer: bell shaped***
38. The normal distribution is a ---------------------------- probability distribution. ***Answer: two parameter.***
39. The -------- of the normal distribution lies at the centre of normal curve. ***Answer: mean***
40. The MGF of Binomial distribution is -------------. ***Answer:*** $(\boldsymbol{q + pe^t})^{\boldsymbol{n}}$
41. The mean of the binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution. ***Answer: q = 4/5, p=1/5, n=100***
42. Define weibull distribution.
43. If 'X' is a uniform variable in [-2,2], find the mean and variance of 'X'. ***Answer: 0,4/3***
44. State the memory less property of exponential distribution.
45. The lifetime of a component measured in hours is Weibull distribution with parameter $\alpha = 0.2, \beta = 0.5$. Find the mean life time of the component. ***Answer: 50 hours***
46. X is a discrete R.V having the p.m.f

X :	-1	0	1
P(X):	k	2k	3k.Find P(X≥0).**Answer : k= 1/6. P(X≥0) = 5/6.**

47. The random variable X has the p.m.f.P(x) = x/15,x=1,2,3,4,5 and =0 elsewhere.find P(1/2<X<5/2/X>1). **Answer :1/7**
48. A dice is thrown 3 times.If getting a '6' is considered a success,find the probability of atleast two successes.**Answer: p = 1/6,q=5/6,n=3,P(atleast two successes) = 2/27**
49. If the p.d.f of a R.V.X is f(x)=x/2 in 0 ≤ x ≤ 2,find P(X>1.5/X >1).**Answer: 0.5833**
50. Comment on the following."The mean of a binomial distribution is 3 and variance is 4"**. Answer:For binomial distribution,Variance < Mean.**

<u>6 MARKS / 12 MARKS</u>

1. A random variable X has the probability function $P(x) = \frac{1}{2^x}, x = 1,2,3, \ldots$Find the MGF, mean and variance.
2. A continuous random variable X has a pdf $f(x) = 3x^2, 0 \le x \le 1$. Find a and b such that
   a. $P(X \le a) = P(X > a)$ *and* b. $P(X > b) = 0.05$.
3. For the distribution defined by the pdf
   $$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & otherwise \end{cases}$$
   Compute the r th moment about the origin. Hence deduce the first four moments about mean.
4. Let X be a random variable with pdf $f(x) = \begin{cases} \frac{1}{3}e^{-x/3}, & x > 0 \\ 0, & otherwise \end{cases}$
   Find a. P (X > 3) b. MGF of X c. E(X) d. Var(X).
5. Six dice are thrown 729 times. How many times do you expect at least 3 dice to show a 5 or 6?
6. Find MGF of binomial distribution. Hence derive mean, variance and standard deviation.
7. A manufacturer of cotton pins knows that 5% of his product is defective. If he sells pins in boxes of 100 and guarantees that not more than 4 pins will be defective. What is the approximate probability that a box will fail to meet the guaranteed quality?

8. Fit a poisson distribution to the following data, which gives the number of yeast cells per square for 400 squares.

No.of cells per square (x)	0	1	2	3	4	5	6	7	8
No. of squares (f)	103	143	98	42	8	4	2	0	0

9. State and prove the memory less property of Geometric distribution.
10. An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives?
11. Find MGF, mean and variance of Uniform distribution.
12. State and prove the memory less property of Exponential distribution.
13. Suppose that the life of an industrial lamp, in thousands of hours, is exponentially distributed with failure rate $\lambda = \frac{1}{3}$. Find the probability that the lamp will last
    a. Longer than its mean life of 3000 hours.
    b. Between 2000 and 3000 hours
    c. For another 1000 hours given that it is operating after 2500 hours.
14. The daily consumption of milk in a city in excess of 20,000 gallons is approximately distributed as a Gamma variable with parameter $= 2$ and $\lambda = \frac{1}{10000}$. The city has a daily stock of 30000 gallons. What is the probability that the stock is insufficient on a particular day?
15. In a certain city, the daily consumption of electric power in millions of kilowatt hours can be treated as a gamma random variable with parameters $v = 3$ and $\lambda = \frac{1}{2}$. If the power plant of this city has a daily capacity of 12 million kilo-watt hours, what is the probability that this power supply is inadequate on any given day?
16. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and Standard deviation of the distribution.
17. In an examination, it is laid down that a student passes if he secures 30% or more marks. He is placed in first, second and third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. If he secures 80% or more marks, he gets distinction. It is noticed from the results that 10% of the students failed and 5% of them obtained distinction. Assuming normal distribution of marks, what percentage of students placed in the second division?
18. A random variable X has a uniform distribution over the interval (-3,3). Find
    a. P[X = 2] b. P[X < 2] c. P[|X| < 2] d. P[|X – 2| < 2] and
    e. Find K such that P [X > K] = 1/3.
19. If X is a normal variable with mean 30 and S.D = 5, then find
    a. P [26 ≤ X ≤ 40] b. P [X ≥ 45] c. P [|X – 30|>5]

UNIT – II – TWO DIMENSIONAL RANDOM VARIABLES

2 MARKS

1. If X and Y are random variables having the joint density function f(x,y) = $\frac{1}{8}$ (6-x-y),0 < x < 2, 2 < y < 4. Find P(X + Y < 3)

   Answer: Given f(x,y) = $\frac{1}{8}$ (6-x-y),0 < x < 2, 2 < y < 4

   P(X + Y < 3) = $\int_2^3 \int_0^{3-y} \frac{1}{8}(6-x-y)dxdy = \frac{1}{8}\left[18y - \frac{6y^2}{2} - \frac{9}{2}y + \frac{6y^2}{4} - \frac{y^3}{6} - \frac{3y^2}{2} + \frac{y^3}{3}\right]$***, y varies from 2 to 3 = 5/24***
2. $E(XY) = E(X)E(Y)$ if X and Y are ------------ variables. ***Answer: Independent***
3. Define Covariance.

   A measure of association between two random variables obtained as the expected value of the product of the two random variables around their means. i.e. $Cov(X,Y) = E(XY) - E(X)E(Y)$.
4. If X and Y are independent, then their covariance is ---------. ***Answer: zero***
5. Say True or False:
   If X and Y are uncorrelated, then they are not necessarily statistically independent.***(TRUE)***
6. Say True or False: The variance of the sum of the random variables equals the sum of the variances if the random variables are uncorrelated. ***Answer: TRUE***
7. Say True or False: Correlation between variables gives the relationship between them.***(TRUE)***
8. Say True or False: Regression between variables gives the relationship between them. ***(TRUE)***
9. Say True or False:Regression between X and Y is the same as that between Y and X. ***Answer: FALSE***
10. Say True or False:Correlation between X and Y can be infinity. ***Answer: FALSE***
11. Find the acute angle between the two lines of regression.

$$\tan\theta = \frac{1-r^2}{r}\left(\frac{\sigma_x\sigma_y}{\sigma_x{}^2+\sigma_y{}^2}\right)$$

12. State Central limit theorem.
13. The regression lines of X on Y and Y on X are $5x - y = 22, 64x - 45y = 24$ respectively. Find the means of X and Y. ***Answer: 6,8.***
14. X and Y are independent random variables with variance 2 and 3.Find the variance of 3X+4Y
    Answer: G.T X and Y are independent RVS with variance 2 and 3.
    Var(X) = 2,var(Y) = 3 .var(3X+4Y) = 9var(X) + 16 var(Y) = 9 x 2 +16 x 3 = **66**
15. The minimum and maximum values of the correlation coefficient are --- and -----. ***Answer: -1 , 1***
16. Two random variables are said to be orthogonal if --------. ***Answer: their correlation is zero.***
17. The following table gives the joint probability distribution of X and Y.Find the (a) marginal density function of X. (b) marginal density function of Y

Y/ X	1	2	3
1	0.1	0.1	0.2
2	0.2	0.3	0.1

Answer: P(y = 1) = 0.4 and P(y = 2) = 0.6. P(x = 1) = 0.3, P(x=2) = 0.4, P(x=3) = 0.3.

18. If X and Y be integer valued random variables with $P(X = m, Y = n) = q^2 p^{m+n-2}$, n,m=1,2,3.....and p + q = 1. Are X and Y independent? ***Answer: yes***

6 MARKS / 12 MARKS

1. Let X and Y be two random variables having the joint probability function f(x,y) = k(x+2y) where X and Y can assume only integer values 0,1 and 2. Find the marginal and conditional distributions.
2. From the following bivariate probability distribution,

Y/X	-1	0	1
0	1 / 15	2 / 15	1 / 15
1	3 / 15	2 / 15	1 / 15
2	2 / 15	1 / 15	2 /1 5

   Find a. Marginal distributions of x and Y b. Conditional distributions
3. If $f(x) = e^{-(x+y)}, 0 \le x, y \le \infty$ is the joint pdf of random variables X and Y, find
   a. P [X < 1] b. P [X > Y] c. P [X + Y ≤ 1]
4. Two continuous random variables X and Y have the joint pdf
   $$f(x,y) = \begin{cases} 6\,(1-x-y), x > 0, y > 0, 0 < x+y < 1 \\ 0, \quad elsewhere \end{cases}$$
   Find the marginal distributions and conditional distributions of X and Y. Hence examine whether X and Y are independent.
5. If X and Y are independent random varaibles each normally distributed with mean zero and variance σ^2, find the density function of $R = \sqrt{X^2+Y^2}$ and $\theta = tan^{-1}(\frac{Y}{X})$.
6. Find Correlation co-efficient between X and Y from the following data:

X	78	89	97	69	59	79	61	61
Y	125	37	156	112	107	136	123	108

7. If X and y are uncorrelated random variables with variances 16 and 9, find the correlation coefficient between x + y and x – y.
8. Find the two lines of regression for the following data.

X	150	152	155	157	160	161	164	166
Y	154	156	158	159	160	162	161	164

9. The two lines of regression are $8x - 10y = -66, 40x - 18y = 214$. The variance of X is 9. Find the mean values of X and the correlation cooeeficient between X and Y.
10. The life time of a certain brand of a tube light may be considered as a random variable with 1200 hours and SD 250 hours. Find the probability using CLT, that the average life time of 60 lights exceeds 1250 hours.
11. A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using CLT, with what probability can we assert that the mean of the sample will not differ from μ = 60 by more than 4.
12. A distribution with unknown mean μ has variance equal to 1.5. Use CLT, to determine how large a sample should be taken from the distribution in order that the probability will be atleast 0.95 that the sample mean will be with in 0.5 of the population mean.

UNIT – III – CLASSIFICATION OF RANDOM PROCESSES
2 MARKS

1. What are all the four types of Stochastic process.
   Answer: ***a. Continuous random process***
   b. Discrete random process
   c. Continuous random sequence
   d. Discrete random sequence.

2. State any two properties of Poisson process.
   Answer: ***a. The Poisson process is not a Markov process.***
   b. The sum of two independent Poisson processes is a Poisson process.
3. Prove that the difference of two independent Poisson processes is not a Poisson process.
   Answer: $X(t) = X_1(t) - X_2(t)$, $E[X(t)] = (\lambda_1 - \lambda_2)t$,$E(X^2(t)] \neq (\lambda_1 - \lambda_2)(t) - (\lambda_1 - \lambda_2)^2 t^2$
4. Define WSS.
   Answer: A random process { X(t)} is called WSS if mean is a constant and the auto correlation depends only on the time difference.
5. Define SSS.
   Answer: A random process is called SSS if all its finite dimensional distributions are invariant under translation of time parameter.
6. Give an example of Markov process.
   Answer: Let X(t) = no.of births upto time 't' so that the sequence $\{X(t), t \in [0, \infty]\}$ ***forms a pure birth process.Then it forms a Markov process since the future is independent of the past given the current state.***
7. Define Markov chain
   Answer: If X(t) is a Markov process which posses Markov property which takes only discrete lues whether t is continuous or discrete is called Markov chain.
8. State the postulates of a poisson process.
   Answer: Let X (t) = no.of times an event A say,occurred upto time 't' so that the sequence $\{X(t), t \in [0, \infty]\}$ ***forms a poisson process with parameter '***λ***' .***
   (i) Events occurring in non-overlapping intervals are independent of eachother
   (ii) $P[X(t) = 1 \text{ for } t \text{ in } (x,x+h)] = \lambda h + o(h)$
   (iii) $P[X(t) = 0 \text{ for } t \text{ in } (x,x+h)] = 1 - \lambda h + o(h)$
   (iv) $P[X(t) = 2 \text{ or move for } t \text{ in } (x,x+h)] = o(h)$
9. What is continuous random sequence ?Give an example
   Answer: If T,the parameter set is discrete and S ,the state space is continuous ,the random process is called a continuous random sequence.Ex: if X_n ***represents the temperature at the end of the*** n^{th} ***hour of a day, then*** $\{X_n, 1 \le n \le 24\}$ ***is a continuous random sequence, since the temperature can take any value in an interval and hence continuous.***
10. What is stochastic matrix?when is it said to be regular?
   Answer: $P_{ij} \geq 0$ ***and*** $\sum_j Pij = 1$ ***for all I then the tpm(transition probability matrix) of a Markov chain is a stochastic matrix. A stochastic matrix 'P' is said to be a regular matrix,if all the entries of*** P^m ***are +ve.***
11. Define irreducible Markov chain?
   Answer: If $P_{ij}(n) \geq 0$ ***for some n and for all i &j,then every state can be reached from other state.When this condition is satisfied,the Markov is said to be irreducible.The tpm of an irreducible chain is an irreducible matrix.***
12. Chapman – Kolomorgow theorem: State Chapman – Kolmorgow theorem
   Answer: If 'p' is the tpm of a homogeneous Markov chain,then the n-step tpm $P^{(n)}$ ***is equal to*** P^n, $\left[P_{ij}^{(n)}\right] = \left[P_{ij}\right]^n$
13. A random process is called Deterministic if -------------
   Answer: all the future values can be predicted from the past observations.
14. The random process is a random variable which is -------------- on time. ***Answer: dependent***
15. The random process at a particular time instant is a -----------------------. ***Answer: random variable***
16. A random process with time averages equal to ensemble averages is called as -------------.
   Answer: Ergodic process
17. Practically, no process is -----------------. ***Answer: SSS***
18. A true SSS process ranges from ----- to ------. ***Answer:*** $-\infty$, ∞

19. Every ergodic process is --------------- process*. Answer: stationary*
20. Say True or False: A stationary process is not necessarily an ergodic process. **(True)**
21. Say True or False: Every SSS is a WSS**. (True)**
22. Say True or False: $X(t_i, s_j)$ is a real number. **(True)**
23. Say True or False: $X(t_i, s)$ is a random variable. **(True)**
24. Say True or False: $X(t, s_j)$ is a sample function **(True)**
25. Say True or False: A WSS is not a second order stationary process**. (False)**
26. Say True or False: A WSS should be first order stationary. **(False)**
27. Say True or False: All stationary process is ergodic. **(False)**
28. Say True or False: All ergodic process is stationary. **(True)**
29. Say True or False: A Markov process has unlimited historical dependency. **(False)**
30. Say True or False: The TPM of a finite state Markov chain is not a square matrix. **(False)**
31. The TPM of a finite state Markov chain takes only-----------------. ***Answer: Non – negative values***
32. All regular Markov chains are -------------. ***Answer: ergodic***
33. The sum of all the elements in any row in the TPM of a finite state Markov chain is ----.
Answer: one
34. Is it a valid $\begin{pmatrix} 0.2 & 0.8 \\ 0.1 & 0.5 \end{pmatrix}$ TPM? ***Answer: No***
35. Poisson process is a --------- random process. ***Answer: Discrete.***
36. Say True or False: The poisson process is an independent increment process with Markov property. ***(True)***
37. Say True or False: The inter arrival time of a Poisson process is also Poisson***. (False)***
38. Say True or False: Poisson process is neither stationary nor Markov***(False)***
39. Say True or False: The Poisson process has a mean 6 and SD 4***.(False)***

6 MARKS /12 MARKS

1. Prove that the difference of two independent Poisson Processes is not a Poisson Process.
2. Show that the random process X(t) = A cos (wt + θ) is wide sense stationary if A and w are constant and θ is uniformly distributed random variable in (0, 2π).
3. Determine whether or not the process X(t) =A sin(μt)+ B cos (μt) is ergodic, if A and B are normally distributed random variables with zero mean and unit variances.
4. Show that if {X(t)} is a WSS process then the output {Y(t)} is a WSS process.
5. A Random Process {X(t),t $\in$ T} has the probability distribution
$P\{X(t) = n\} = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2 \ldots \\ \frac{at}{1+at}, n = 0 \end{cases}$ Show that the process is evolutionary.
6. The transition probability matrix of a Markov chain {X_n}, three states 1,2 and 3 is
P = $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the distribution is $P^{(0)} = (0.7,0.2,0.1)$. Find
   i. P{X_2 =3}
   ii. P{X_3 = 2, X_2 =3, X_0 =2}.
7. Let the two random process X(t) and Y(t) be defined as X(t) = A cos wt +B sin wt, Y(t) =B cos wt – A sin wt where A and B are random variables and w is a constant. If A and B are uncorrelated random variable with zero mean and equal variance prove that X(t) and Y(t) are jointly WSS.
8. A stochastic process is described by X(t) = A sint +B cost where A and B are independent random variables with zero means and equal standard deviation. Show that the process is stationary of the second order.
9. Three boys A,B,C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to a show that the process is Markovian. Find the transition matrix and classify the states.
10. Let $\{X_n \ , n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and with the transition matrix $\begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$

UNIT – IV – CORRELATION AND SPECTRAL DENSITIES

2 MARKS

1. Find the mean and variance of the process given that the ACF $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$
Answer: Mean = 5, Variance = 4

2. Find the mean of the stationary process {X(t)} whose autocorrelation function $R(\tau) = \frac{25\tau^2+36}{6.25\tau^2+4}$
Answer: Mean = 2
3. The ACF is an ---- function. ***Answer: Even***
4. The ACF is ----- at the origin. ***Answer: maximum***
5. The CCF is not an ----- function. ***Answer: Even***
6. The CCF of two random processes does not have a maximum value at --------. ***Answer: the origin.***
7. The significance of PSD is --------------. ***Answer: it gives power / band width.***
8. The ACF of a process $R(\tau)$ at $\tau = 0$ gives ------------------ of the signal. ***Answer: Mean square value***
9. Weiner – Khinchine relation states that ---------------------.
Answer: ACF and PSD of a random process form a Fourier transform pair.
10. The ACF is a measure of ---------------. ***Answer: Inter – dependence of two random processes.***
11. If the random process is periodic, then its ACF is ------------------. ***Answer: Periodic***
12. If two independent random process are of zero mean, then their correlation is ------. ***Answer: Zero***
13. Two independent random process will have their cross correlation as ----- of individual means.
Answer: product.
14. The PSD of a WSS is always -----------. ***Answer: non – negative***
15. Cross correlation and cross PSD form a -------------. ***Answer: Fourier transform pair***
16. If X(t) and Y(t) are orthogonal process, then their cross correlation is ------ and their cross PSD is ---------
Answer: zero, zero
17. The auto correlation of a random process $R(\tau)$ at $\tau = 0$ is equal to its -------. **Answer**: **second moment**
18. Say True or False: The auto correlation function of the output of a linear system is symmetric function. **(True)**
19. Say True or False: The cross correlation of the input and output of a linear system is symmetric function. ***(False)***
20. Define spectral density.
Answer: Let {X(t),t≥0) be astationary time series with E[X(t)] = 0 and covariance function R(t – s) = E[X(t)X(s)] and let F(x) be a real, never decreasing and bounded function of x with dF(x) = f(x) dx. R(t) is non-negative definite then F(w) = $\frac{1}{2\pi}\int_{-\infty}^{\infty} R(\tau)e^{-i\tau\omega}\, d\tau$
21. Define cross – spectral density.
Answer: Cross – spectral density of the two jointly WSS continuous – time process {X(t),y(t)} is defined as the Fourier transform of the Cross – Correlation $R_{xy}(z)$ given by
$S_{xy} = \int_{-\infty}^{\infty} R_{xy}(z)e^{-j2\pi fz}\, dz$

6 MARKS /12 MARKS

1. Find the Power spectral density of the random process, if its autocorrelation function is given by $R_{XX}(\tau) = e^{-\alpha|\tau|}\cos\beta\tau$.
2. State & Prove Wiener Khintchine theorem.
3. Find the power spectral density of a random signal with autocorrelation function a $e^{-b|\tau|}$
4. The power spectral density of a wide sense stationary process given by
$S(w) = \begin{cases} \frac{b}{a}(a-|w|) & ,|w| \le a \\ 0 & ,|w| > a \end{cases}$. Find the auto correlation of the process.
5. The power spectrum of a WSS X ={X (t)} is given by $F_x(w) = \frac{1}{(1+w^2)^2}$. Find the auto correlation function.
6. If the WSS process X(t) is given by $X(t) = 10\cos(100t + \varphi)$ *where* φ is uniformly distributed in $(-\pi, \pi)$. Prove that X is ergodic with respect to the auto correlation function.
7. Show that the random process $X(t) = \sin(wt + ⍰)$ *where w is a constant and* ⍰ is random variable uniformly distributed in $(0, 2\pi)$ is
   i. First order stationary ii. Also find the auto correlation function of the process.
8. The auto correlation of the random binary transmission is given by $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, |\tau| < T \\ 0, |\tau| > T \end{cases}$. Find the power spectrum.
9. The cross power spectrum of the process X(t) is $S_{XY}(w) = \begin{cases} a + \frac{ibw}{w}, -w < w < w \\ 0, \quad o.w \end{cases}$
W >0 , a and b are real constants. Find the cross correlation function.
10. Find the power spectral density of aWSS process with autocorrelation function $R(T) = e^{-\alpha\tau^2}$.
11. The auto correlation function of an periodic power signal is $R_{XX}(\tau) = e^{-\tau^2\sigma^2/2}$. Find the power spectral density and normalized average power content of the signal.

UNIT – V – LINEAR SYSTEMS WITH RANDOM INPUTS

2 MARKS

1. If the input for an LTI system is Gaussian process, then the output is ----------
   Answer: Gaussian
2. A ------- is a functional relationship between the input and the output. ***Answer: System***
3. Describe a linear system with an random input.
   Answer: We assume that x(t) represents a sample function of a random process {X(t)}. The system produces an output or response y(t) and the ensemble of the output functions forms a random process {Y(t)}. The process {Y(t)} can be considered as the output of the system or transformation 'f' with {X(t)} as the input, the system is completely specified by the operator 'f'.
4. Define Time – invariant system.
   Answer: Let Y(t) = f [X(t)]. If Y(t+h) = f [X(t+h)], then 'f' is called a time – invariant system.
5. Define memory less system.
   Answer: If the output Y(t) at a given time t = to depends only on X(to) and not on any other past or future values of X(t), then the system 'f' is called Memoryless system.
6. *What is a stable system?*
   Answer: A linear time invariant system is said to be stable if ite response to any bounded input is bounded.
7. In white noise, the PSD contains all frequencies in ----- amount. ***Answer: equal***
8. State the properties of a linear filter.
   Answer: Let {X_1(t)} and X_2(t) be any two processes and 'a' and 'b' be two constants.If l is a linear filter then L[a X_1(t) + b X_2(t) = a L(X_1(t)[+ b L(X_2(t)]

6 MARKS /12 MARKS

1. A signal $x(t) = u(t)e^{-\alpha t}$ is applied to a network having an impulse response $h(t) = W\, u(t)e^{-Wt}$. Here α *and* W are real positive constants and $u(.)$ is the unit-step function. Find the system's response.
2. A signal $x(t) = u(t)e^{-\alpha t}$ is applied to a network having an impulse response $h(t) = W\, u(t)e^{-Wt}$. Here α *and* W are real positive constants and $u(.)$ is the unit-step function. Find the spectrum $Y(\omega)$ of the response.
3. A rectangular pulse of amplitude A and duration T, defined by $x(t) = \begin{cases} A\,, 0 < t < T \\ 0, else\ where \end{cases}$
   is applied to the system of problem 1. Find the time response $y(t)$ and Sketch your response for $W = \frac{\pi}{T}$ *and* $W = \frac{2\pi}{T}$
4. A network has the transfer function $H(\omega) = \frac{2\, e^{j\omega/20}}{(20+j\omega)^3}$
   a) Determine and sketch its impulse function.
   b) Is the network physically reliable?
5. A random process X(t) has an autocorrelation function $R_{XX}(\tau) = A^2 + Be^{-|\tau|}$ where A and B are positive constants. Find the mean value of the response of a system having an impulse response
   $h(t) = \begin{cases} e^{-Wt}\,, 0 < t \\ 0, t < 0 \end{cases}$ where W is a real positive constant, for which X(t) is its input.
6. Two separate systems have impulse responses $h_1(t)$ and $h_2(t)$. A process $X_1(t)$ is applied to the first system and its response is $Y_1(t)$. Similarly, a process X2(t) invokes a response $Y_2(t)$ from the second system. Find the cross –correlation function of $Y_1(t)$ and $Y_2(t)$ in terms of $h_1(t)$,$h_2(t)$, and the cross –correlation function of $X_1(t)$ and $X_2(t)$. Assume $X_1(t)$ and $X_2(t)$ are jointly wide sense stationary.
7. Two systems are cascaded. A random process X(t) is applied to the input of the first system that has impulse response W(t) is the input to the second system having impulse response $h_1(t)$; its response W(t) is the input to the second system having impulse response $h_2(t)$. The second system's output is Y(t). Find the cross –correlation function of W(t) and Y(t) in terms of h_1 (t)and $h_2(t)$, and the autocorrelation function of Y(t) if X(t) is wide sense stationary.
8. A random process X(t) having autocorrelation function $R_{XX}(\tau) = Pe^{-\alpha|\tau|}$ where P and α are real positive constants, is applied to the input of a system with impulse response
   $h(t) = \begin{cases} We^{-Wt}, 0 < t \\ 0, t < 0 \end{cases}$ where W is a real positive constant. Find the autocorrelation function of the network's response Y(t).
9. White noise with power density $N_0/2$ is applied to a network with impulse response
   $h(t) = u(t)Wt\exp(-Wt)$ *where* $W > 0$ is a constant. Find the cross –correlation of the input and output.

10. A random process X(t) is applied to a network with impulse response $h(t) = u(t)\exp(-bt)$ *where* $b > 0$ is a constant. The cross- correlation of X(t0 with the output Y(t) is known to have the same form: $R_{XY}(\tau) = u(\tau)\tau\exp(-b\tau)$

   a) Find the autocorrelation of Y(t).
   b) What is the average power in Y(t)?

11. A circuit has an impulse response given by $h(t) = \begin{cases} \frac{1}{T}, & 0 \leq t \leq T \\ 0, o.w \end{cases}$ Evaluate $S_{YY}(w)$ in terms of $S_{XX}(w)$.
