K.S.R COLLEGE OF ENGINEERING : TIRUCHENGODE – 637 215.

18MA242 – Applied Mathematics QUESTION BANK.

UNIT I - LAPLACE TRANSFORMATION

PART - A

1. Find $L[t\sin 2t]$.

   Solution: $L\{t\, f(t)\} = -\frac{d}{ds}L\{f(t)\} = -\frac{d}{ds}L\{\sin 2t\} = -\frac{d}{ds}\left(\frac{4}{s^2+4}\right) = \frac{4s}{(s^2+4)^2}$

2. Find $L^{-1}\left[\frac{1}{(s-3)^2}\right]$.

   Solution: $L^{-1}\{F(s-a)\} = e^{at}L^{-1}\{f(t)\} = e^{3t}L^{-1}\left[\frac{1}{s^2}\right] = e^{3t}t$

3. Find $L\{te^{-2t}\cos 2t\}$.

   Solution: $L\{tf(t)\} = -\frac{d}{ds}L\{f(t)\} = -\frac{d}{ds}L\{\cos 2t\} = -\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{4-s^2}{(s^2+4)^2}$

   $L\{e^{-at}f(t)\} = F(s+a) = F(s+2) = \frac{4-(s+2)^2}{[(s+2)^2+4]^2}$

4. Prove that $\int_0^\infty te^{-3t}\sin t\,dt = \frac{3}{50}$.

   Solution: Comparing the given integral with LT we get, $f(t) = t\sin t$ and $s = 3$

   $LHS = L\{t\sin t\} = -\frac{d}{ds}\left(\frac{1}{s^2+1}\right) = \frac{2s}{(s^2+1)^2}$ put $s = 3$ we get $LHS = \frac{3}{50} = RHS$

5. Using Laplace transform of derivatives find $L\{e^{-at}\}$.

   Solution: $L\{f'(t)\} = sL\{f(t)\} - f(0)$ here $f(t) = e^{-at}; f'(t) = -ae^{-at}; f(0) = 1$

   $L\{-ae^{-at}\} = sL\{e^{-at}\} - 1$ implies $-aL\{e^{-at}\} = sL\{e^{-at}\} - 1$

   $L\{e^{-at}\}(s+a) = 1$ implies that $L\{e^{-at}\} = \frac{1}{s+a}$

6. Verify the initial value theorem for $f(t) = 3 + 4\cos 2t$.

   Solution: $\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$; $LHS = 3 + 4\cos 0 = 7$; $F(s) = L\{3+4\cos 2t\}$

   $RHS = \lim_{s\to\infty} sL\{3+4\cos 2t\} = \lim_{s\to\infty} s\left(\frac{3}{s}+\frac{4s}{s^2+4}\right) = \lim_{s\to\infty}\left(3+\frac{4}{1+\frac{4}{s^2}}\right) = 3+4 = 7$

7. Find the inverse Laplace transform of $\frac{1}{(s+2)^4}$.

   Solution: $L^{-1}\{F(s+a)\} = e^{-at}L^{-1}\{f(t)\} = e^{-2t}L^{-1}\left[\frac{1}{s^4}\right] = \frac{e^{2t}t^3}{6}$

8. State the conditions under which the Laplace transform of $f(t)$ exists.

   - $f(t)$ should be continuous or piece – wise continuous in the given closed interval $[a, b]$ where $a > 0$.
   - $f(t)$ should be of exponential order.

9. State and prove first shifting property of Laplace transform.

   Statement: If $L\{f(t)\} = F(s)$ then $L\{e^{at}f(t)\} = F(s-a)$

   Proof: $L\{e^{at}f(t)\} = \int_0^\infty e^{-st}[e^{at}f(t)]dt = \int_0^\infty e^{-(s-a)t}f(t)\,dt = F(s-a)$ if $s > a$

10. Find $L[e^{-t}\sin 3t]$.

    Solution: $L\{e^{-at}f(t)\} = F(s+a) = F(s+1)$; $L\{\sin 3t\} = \frac{3}{s^2+9}$ put $s = 1$; $= \frac{3}{(s+1)^2+9}$

11. Find $L[e^{-2t}\cos 3t]$.

    Solution: $L\{e^{-at}f(t)\} = F(s+a) = F(s+2)$; $L\{\cos 2t\} = \frac{s}{s^2+4}$ put $s = 2$; $= \frac{s+2}{(s+2)^2+9}$

12. *If $L\{f(t)\} = F(s)$, find $L\{f(at)\}$.*

Solution: $L\{f(at)\} = \int_0^\infty e^{-st} f(at)]dt = \int_0^\infty e^{-\frac{sT}{a}} f(T)\frac{dT}{a} = \frac{1}{a}F\left(\frac{s}{a}\right)$ *if* $a \neq 0$

13. *State the convolution theorem of Laplace transform.*

Solution: *If* $f(t)$ *and* $g(t)$ *are Laplace transformable, then* $L\{f(t) * g(t)\} = L\{f(t)\}.L\{g(t)\}$

14. *If $f(t)$ is a periodic function with period p, what is its Laplace transform?*

Solution: $L\{f(t)\} = \frac{1}{1-e^{-ps}}\int_0^p e^{-st} f(t)dt$

15. *If* $L\{f(t)\} = \frac{s+2}{s^2+4}$, *find the value of* $\int_0^\infty f(t)dt$.

Solution: $\therefore f(t) = L^{-1}\left(\frac{s}{s^2+2^2}\right) + L^{-1}\left(\frac{2}{s^2+2^2}\right) = cos2t + sin2t$

Now, $\int_0^\infty (cos2t + sin2t)dt = \left(\frac{sin2t - cos2t}{2}\right) = 0 - 1 = -1$

16. *Find* $L[t2cos3t]$.

Solution: $L\{t^2 f(t)\} = \frac{d^2}{ds^2}L\{f(t)\} = \frac{d^2}{ds^2}L\{cos3t\} = \frac{d^2}{ds^2}\left(\frac{s}{s^2+9}\right) = \frac{2s(s^2-27)}{(s^2+9)^3}$

17. *Find the inverse Laplace transform of* $\frac{1}{s(s-a)}$.

Solution: $L^{-1}\left[\frac{F(s)}{s}\right] = \int_0^t L^{-1}\{F(s)\}dt = \int_0^t L^{-1}\left[\frac{1}{s-a}\right]dt = \int_0^t e^{at}\,dt = \frac{e^{at}}{a} - \frac{1}{a}$

18. *Find $f(t)$ if* $L\{f(t)\} = \frac{s}{(s+2)^2}$.

Solution: $L^{-1}\{sF(s)\} = \frac{d}{dt}L^{-1}\{F(s)\} = \frac{d}{dt}L^{-1}\left\{\frac{1}{(s+2)^2}\right\} = \frac{d}{dt}(e^{-2t}t) = e^{-2t}(1-2t)$

19. *Find the inverse Laplace transform of* $log\left(\frac{s+1}{s-1}\right)$.

Solution: $\therefore L\{f(t)\} = log\left(\frac{s+1}{s-1}\right) = log(s+1) - log(s-1)$

$L\{tf(t)\} = -\frac{d}{ds}[log(s+1) - log(s-1)] = \frac{1}{s-1} - \frac{1}{s+1} = \frac{2}{s^2-1}$

$tf(t) = L^{-1}\left(\frac{2}{s^2-1}\right) = 2sinht;\quad f(t) = \frac{2sinht}{t}$

20. *Solve using Laplace transform* $\frac{dy}{dt} + y = e^{-t}$ *given that* $y(0) = 0$.

Solution: $L(y') + L(y) = L(e^{-t});\ (s+1)L(y) = \frac{1}{s+1}$ *since* $y(0) = 0$

$L(y) = \frac{1}{(s+1)^2}\ ;\ y = L^{-1}\left(\frac{1}{(s+1)^2}\right) = te^{-t}$

21. *Does Laplace transform of* $\frac{cosat}{t}$ *exist? Justify.*

Solution: *No, since* $\lim_{t\to\infty} e^{-st}\frac{cosat}{t} = \infty$

22. *Find the inverse Laplace transform of* $cot-1s$.

Solution: $L\{tf(t)\} = -\frac{d}{ds}F(s);\ \ tf(t) = L^{-1}\left\{-\frac{d}{ds}F(s)\right\} = L^{-1}\left[-\frac{d}{ds}(cot^{-1}s)\right] = L^{-1}\left(\frac{1}{1+s^2}\right) = sint$

$\therefore f(t) = \frac{sint}{t}$

23. *Find* $L(t2\ 2t)$.

Solution: $L\{t^2 2^t\} = L\{t^2 e^{tlog2}\} = F(s - log2)$

$F(s) = L\{t^2\} = \frac{2}{s^3}$ $\quad \therefore F(s-\log 2) = \frac{2}{(s-\log 2)^3}$

24. *Obtain the Laplace transform of sin 2t – 2t cos 2t in the simplified form.*

Solution: $L\{\sin 2t - 2t\cos 2t\} = \frac{2}{s^2+4} - \frac{8-2s^2}{(s^2+4)^2} = \frac{2s^2+8-8+2s^2}{(s^2+4)^2} = \left(\frac{2s}{s^2+4}\right)^2$

25. *Give an example for a function which has Laplace transform but it is not continuous.*

Solution: $f(t) = t^{-\frac{1}{2}}$ since $\lim_{t\to 0+} t^{-\frac{1}{2}} = \infty$

PART - B

1. Find the Laplace transform of $e^{3t}(t\cos 2t)$ and $\frac{1-e^{-2t}}{t}$.
2. Solve the differential equation, using Laplace transform $y'' + 4y' + 4y = e^{-t}$ given that y(0) = 0 and $y'(0) = 0$.
3. Find the Laplace transform of a triangular wave function f(t) = t, 0<t<Π
   2Π - t, Π<t<2Π where f(t+2Π) = f(t).
4. Using convolution theorem find the Laplace inverse of $\frac{s+2}{(s^2+4s+13)^2}$.
5. i)Find L$\left(\frac{e^{-at}-e^{-bt}}{t}\right)$. ii) Find L^{-1}(log$\left(\frac{s^2+a^2}{s^2+b^2}\right)$).
6. Using convolution theorem find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$.
7. Using Laplace transform solve $y'' + 2y' - 3y = \sin t$ given that y(0) = 0 and $y'(0) = 0$.
8. Find the Laplace transform of $\int_0^t \frac{e^{-t}\sin t}{t}dt$
9. Find the Laplace transform of f(t) = k, $0 \le t \le a$
   -k, a$\le t \le 2a$ and f(t+2a) = f(t).
10. Verify the final value theorem for the function f(t) = $L^{-1}\left(\frac{1}{s(s+2)^2}\right)$.
11. Find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$, using convolution theorem.
12. Solve $\frac{dy}{dt} + 3t + 2\int_0^t y\,dt$ = t given y(0) = 1.
13. Using convolution theorem find $L^{-1}\left(\frac{4}{(s^2+2s+5)^2}\right)$.
14. Solve by Laplace transform method $y'' + 5y' + 6y = 2, y(0) = y'(0) = 0$.
15. Find the inverse Laplace transform of $\frac{s+2}{(s+3)(s^2+4)}$ and log$\left(\frac{s-5}{s^2+9}\right)$.
16. Using Laplace transform solve $y'' - 2y' + y = e^t$ given that y(0) = 2and $y'(0) = 0$.
17. Find the Laplace transform of f(t) = cost, $0 < t < \pi$,
    =0, $\pi < t < 2\pi$ and f(t+2π) = f(t).
18. Find the inverse Laplace transform of $\frac{s^2}{(s^2+1)^2}$ using convolution theorem.

19. Find the Laplace transform of t e^{2t}sin3t.

20. (i)Find the inverse Laplace transform of $\frac{1}{s^3(s+5)}$ using convolution theorem.

(ii)Verify initial value theorem for the function (1 + e^{-2t}).

21. (i)Find $L\left\{e^{-t}\int_0^t \frac{\sin t}{t}dt\right\}$. (ii)Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\right\}$.

22. (i) Find $L\left(\frac{e^{at}-\cos bt}{t}\right)$. (ii)Find L (sint u_π (t)), where u_π (t) is unit step function.

23. Evaluate $\int_0^\infty \frac{e^{-2t}\sin^2 t}{t}dt$.

24. (i) Evaluate using Laplace transform : $\int_0^\infty te^{-2t}\sin 3t\,dt$. (ii) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$.

25. Solve using Laplace transforms : y'' +4y' +4y = t e^{-t}, y (0) = 0,y' (0) = -1.

UNIT - II COMPLEX VARIABLES

PART – A

1. State the orthogonal property of an analytic function.
Solution: If $f(z)=u+iv$ is an analytic function of $z=x+iy$, then the curves $u(x,y)=c_1$ and $v(x,y)=c_2$ cut orthogonally.
2. Find the critical points for the transformation $w^2=(z-\alpha)(z-\beta)$.
Solution: Critical point at $\frac{dw}{dz}=0 \Rightarrow w\frac{dw}{dz}=z-\frac{1}{2}(\alpha+\beta)=0 \Rightarrow z=\frac{\alpha+\beta}{2}$.
Critical points occur at $w=0$ also $(z-\alpha)(z-\beta)=0$
The critical points are $z=\alpha,\beta,\frac{\alpha+\beta}{2}$
3. What are the sufficient conditions for a function $w=f(z)=u+iv$ to be analytic?
Solution: The continuous single-valued function $f(z)=u+iv$ is analytic in a region R, if the four derivatives u_x, v_x, u_y and v_y exist, and are continuous and satisfy the C-R equation at each point of R.
4. Define bilinear transformation.
Solution: The transformation $w=\frac{az+b}{cz+d}$ where a,b,c and d are complex constants and $ad-bc\neq 0$.
This transformation is also called as mobius transformation.
5. Write down the formula for finding an analytic function f(z) = u+iv, whenever the real part is given, by using the Milne Thomson method.
Solution: $f(z)=\int[\phi_1(z,0)-i\phi_2(z,0)]dz+c$
6. Verify the function $u(x,y)=e^x\sin y$ is harmonic or not.
Solution: $u_x=e^x\sin y; u_{xx}=e^x\sin y$ and $u_y=e^x\cos y; u_{yy}=-e^x\sin y$.
$u_{xx}+u_{yy}=0 . \Rightarrow u$ Is harmonic.
7. If u and v are harmonic function then can we say that $u+iv$ is analytic?
Solution: Yes. But it is not always. Example: $u=x$ and $v=-y$.
8. Verify the function $u(x,y)=\log\sqrt{(x^2+y^2)}$ is harmonic or not?
Solution: $u_x=\frac{1}{2}\left(\frac{2x}{x^2+y^2}\right)$; $u_{xx}=\frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} \Rightarrow \frac{y^2-x^2}{(x^2+y^2)^2}$

$$u_y = \frac{1}{2}\left(\frac{2y}{x^2+y^2}\right) \quad ; \qquad u_{yy} = \frac{(x^2+y^2)-2y^2}{(x^2+y^2)^2} \Rightarrow \frac{x^2-y^2}{(x^2+y^2)^2} \qquad u_{xx}+u_{yy} = 0.$$

$\therefore$ u Is harmonic.

9. Find the image of | z - 2i |= 2 under the mapping $\omega = \frac{1}{z}$.

Solution: $|z-2i| = 2 \Rightarrow |x+iy-2i| = 2 \Rightarrow x^2+y^2-4y = 0. \to (1)$

Now $w = u+iv \Rightarrow z = \frac{1}{u+iv} \Rightarrow x+iy = \frac{u-iv}{u^2+v^2}$.

Equating real part and imaginary part, and substituting in (1), we get
$1+4v = 0$ Which is a straight line in $w-plane$.

10. When do we say $w = f(z)$ is a conformal mapping?

Solution: The mapping $w = f(z)$ is conformal if $f(z)$ is analytic at each point and $f'(z)$ is not zero.

11. Define conformal mapping.

A mapping or transformation which preserves angles both in magnitude and direction between every pair of curves through a point is called a conformal mapping.

12. Prove that $w = z^2$ is analytic and hence find $\frac{dw}{dz}$.

Solution: $z^2 = x^2-y^2+2ixy = u+iv \Rightarrow u = x^2-y^2$ and $v = 2xy$.

$u_x = v_y = 2x$ And $-u_y = v_x = 2y$.Hence $\omega = z^2$ is analytic.

$$\frac{dw}{dz} = u_x + iv_x \Rightarrow 2(x+iy) = 2z$$

13. If u + iv is analytic , show that v – iu is also analytic.

Solution: $u+iv$ is analytic $\Rightarrow u_x = v_y$ and $u_y = -v_x$

For $v-iu, \Rightarrow$ $u_y = -v_x$ $\quad v_y = -(-u_x) \Rightarrow u_x = v_y$

$\therefore$ C-R equation are satisfied for $v-iu$ $\Rightarrow$ $v-iu$ is analytic.

14. Is f(z) = z^3 analytic ? Justify.

Solution: $z^3 = (x^3-3xy^2)+i(3x^2y - y^3) = u+iv$; $u = x^3-3xy^2$ and $v = 3x^2y-y^3$

$\therefore u_x = 3x^2-3y^2$; $v_x = 6xy$; $u_y = -6xy$; $v_y = 3x^2-3y^2$.

$\therefore$ C-R equation is satisfied. Hence z^3 is analytic.

15. Prove that $z\bar{z}$ is nowhere analytic.

Solution: $z\bar{z} = x^2+y^2$. C-R equations are not satisfied.

Further an analytic function is independent of $\bar{z}$. $\therefore$ $z\bar{z}$ is nowhere analytic.

16. Verify whether $w = \bar{z}$ is analytic or not.

17. **Solution:** $w = \bar{z} = x-iy$, $u = x$ and $v = -y \Rightarrow u_x = 1,\ v_x = 0$; $u_y = 0, v_y = -1$.

$\Rightarrow u_x \neq v_y, u_y \neq -v_x \qquad \therefore w = \bar{z}$ *is not analytic.*

18. Examine whether the function xy^2 can be real part of an analytic function.

Solution: $u = xy^2 \Rightarrow u_x = y^2,\ u_{xx} = 0$; $u_y = 2xy,\ u_{yy} = 2x \Rightarrow u_{xx}+u_{yy} \neq 0.$

So xy^2 cannot be real part of an analytic function.

19. Find the image of the circle |z| = 2 under the transformation $w = 3z$.

Solution: $w = 3z \Rightarrow u + iv = 3x + i3y$

$u = 3x$ and $v = 3y$ and $|z| = 2 \Rightarrow x^2 + y^2 = 4$.. $\therefore u^2 + v^2 = 36$ *is the image.*

20. What is the image of the line x = k under the transformation $w = \frac{1}{z}$.

Solution: $z = \frac{1}{w} \Rightarrow x + iy = \frac{u - iv}{u^2 + v^2}$; $x = \frac{u}{u^2 + v^2}$, $x = k \Rightarrow k(u^2 + v^2) - u = 0$

$\therefore u^2 + v^2 - \frac{u}{k} = 0$

21. State the Cauchy-Riemann equation in polar Co-ordinates.

Solution: $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

22. Say True or False: The function $w = \log z$ is analytic everywhere in the complex plane.

Solution: FALSE $\frac{dw}{dz} = \frac{1}{z}$ at $z = 0$, the function ceases to be analytic.

23. Say True or False: The mapping $w = z^2$ is not conformal at $z = 0$.

Solution: True. $\frac{dw}{dz} = 2z = 0$ at $z = 0$. At $z = 0$, $w = z^2$ is not conformal.

24. The transformation $w = cz$ is known as ………………

(a)rotation (b) reflection (c)translation

(d)Magnification (e) rotation and magnification

Solution: If c is real constant, then it is magnification.

If c is complex constant, then it is rotation and magnification.

25. Find the points at which the transformation $w = \sin z$ is not conformal.

Solution: $\frac{dw}{dz} = \cos z$, $\cos z = 0$ At $z = \frac{\pi}{2}; \frac{3\pi}{2}; \ldots$

26. What is invariant point in a mapping?

Solution: T he fixed points (invariant points) of the transformation are such that the image of z is itself. The invariant points of the transformation $w = f(z)$ is given by the solution of $z = f(z)$.

27. The invariant point of the transformation $w = \frac{1}{z - 2i}$ is……………………

Solution: The invariant points are given by $z = \frac{1}{z - 2i}$,

$$z^2 - 2iz - 1 = 0 \Rightarrow z = \frac{2i \pm \sqrt{-4 + 4}}{2} \Rightarrow z = i$$

28. Find the fixed point of the transformation $w = \frac{z + 1}{2z + 1}$.

Solution:

$$z = \frac{z + 1}{2z + 1} \Rightarrow 2z^2 + z = z + 1 \Rightarrow z^2 = \frac{1}{2} \Rightarrow z = \pm\frac{1}{\sqrt{2}} \quad \textit{are the invariant points.}$$

29. Write the cross ratio of the points z_1, z_2, z_3 and z_4.

Solution: $\frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$ is the cross ratio of the points z_1, z_2, z_3 and z_4.

PART – B

1. If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$.
2. Show that the transformation $\omega=\frac{1}{z}$ transforms all circles and straight lines in the z-plane into circles or straight lines in the ω - plane.
3. Show that under the mapping $\omega=\frac{i-z}{i+z}$, the image of the circle $x^2+y^2<1$ is the entire half of the ω - plane to the right of the imaginary axis.
4. Show that $v=e^x(x\cos y-y\sin y)$ is a harmonic function. Find the corresponding analytic function f(z).
5. Show that the function $v=e^x(x\cos y+y\sin y)$ is harmonic and find the corresponding analytic function f(z) = u+iv.
6. Discuss the conformal mapping $\omega=z^2$.
7. If f(z) = u+iv is an analytic function of z , then prove that $\nabla^2[\log|f'(z)|]=0$.
8. Find the real part of the analytic function whose imaginary part is $e^{-x}(2xy\cos y+(y^2-x^2)\sin y)$.Construct the analytic function.
9. Find the bilinear transformation which maps the points 1,i,-1 on to the points 0,1,∞.Show that the transformation maps the interior of the unit circle of the z- plane onto the upper half of the ω - plane.
10. If f(z) = u+iv is analytic, find f(z) if the real part is given by $u=\frac{\sin 2x}{\cos 2x+\cosh 2y}$.
11. Find the image of the region bounded by the lines x = 0,y = 0 and x+y = 1 in the z- plane by the mapping $\omega=ze^{i\pi/4}$.
12. Prove that the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic. Find the conjugate harmonic function V and the corresponding analytic function f(z).
13. Find the bilinear transformation that maps the points 1+i ,- i, 2-i at the z-plane in to the points 0,1,i of the ω - plane.
14. If f(z) = u+iv is analytic, find f(z) if the real part is given by $u=\frac{\sin 2x}{\cosh 2y-\cos 2x}$.
15. Find the bilinear transformation that maps the points z= -1, 0 , 1 in the z-plane in to the points ω = 0,i,3i in the ω - plane.
16. Show that $v=e^{2x}(y\cos 2y+x\sin 2y)$ is harmonic and find the corresponding analytic function f(z) = u+iv.
17. Find the bilinear transformation that maps the points 0,1,∞ of the z-plane in to the points i,1 ,-i of the ω - plane.
18. Derive the necessary conditions for a function f(z0 = u(x,y) + i v(x,y) to be analytic at a point domain R.
19. Find the bilinear transformation that maps the points ∞,i,0 of the z-plane in to the points 0 , i, ∞ of the ω - plane.
20. If u = log(x^2+y^2), find v and f(z) such that f(z) = u+iv is analytic.
21. Determine the region of the ω - plane into which the first quadrant of z- plane mapped by the transformation ω = z^2.
22. Construct the analytic function f(z) = u+iv given that $2u+3v=e^x(\cos y-\sin y)$.
23. Find the bilinear transformation that maps z = (1,i,-1) into ω =(2,i,-2).
24. If u = x^2-y^2 and $v=\frac{-y}{x^2+y^2}$, prove that both u and v satisfy Laplace equation , but that u+iv is not a regular function of z.

25. Obtain the image of |z-2i| = 2, under the transformation $\omega = \frac{1}{z}$.
26. Find the bilinear transformation that maps z = (1,i,-1) into ω =(i,0,-i).Hence find the image of |z|<1.
27. Find the bilinear transformation that maps z = $(0,1,\infty)$ into ω =(-5,-1,3).What are the invariant points in this transformation?
28. Draw the image of the square whose vertices are at (0,0);(1,0);(1,1);(0,1), in the z-plane, under the transformation ω = (1+i)z. What has this transformation done to the original square?

UNIT – III MULTIPLE INTEGRALS

PART - A

1. Find the value of $\int_0^2\int_0^{x^2} e^{y/x}\,dydx$.

Answer: I = $\int_0^2\int_0^{x^2} e^{y/x}\,dydx = \int\left(\frac{1}{x}e^{y/x}\right)_0^{x^2} dx = \int_0^2 \frac{1}{x}(e^x - 1)dx = \left[\frac{1}{x}(e^x - x) - \left(e^x - x^2/2\right)\right]_0^2 = e^2 - 1$.

2. Evaluate $\int_0^a\int_0^{\sqrt{ay}} xy\,dx\,dy$ Answer: I = $\int_0^a\int_0^{\sqrt{ay}} xydxdy = \int_0^a \frac{a}{2}y^2dy = \frac{a^4}{6}$.

3. Evaluate $\iint_R ydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

Answer: I = $\iint_R ydxdy = \int_0^a\int_0^{\sqrt{a^2-x^2}} ydydx = \frac{1}{2}\int_0^a \left(a^2 - x^2\right)dx = \frac{a^3}{3}$.

5.Evaluate $\iint dxdy$ over the region bounded by x = 0; x = 2; y = 0; y = 2.

Answer: I = $\int_0^2\int_0^2 dxdy = 2\int_0^2 dy = 4$.

6.Change the order of integration in $\int_{-a}^a\int_0^{\sqrt{a^2-y^2}} xdxdy$.

Answer: I = $\int_{-a}^a\int_0^{\sqrt{a^2-y^2}} xdxdy$. y varies from $-\sqrt{a^2-x^2}$ to $\sqrt{a^2-x^2}$; x varies from 0 to a

I = $\int_{-a}^a\int_0^{\sqrt{a^2-y^2}} xdxdy. = \int_0^a\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} xdxdy$.

7. Change the order of integration in $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$.

Answer: I = $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$. x varies from $\frac{y^2}{4a}$ to $2\sqrt{ay}$; y varies from 0 to $4a$. I = $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx. = \int_0^{4a}\int_{y^2/4a}^{2\sqrt{ay}} xydxdy$

8.Change the order of integration in $\int_0^1\int_0^{1-x} g(x,y)dydx$.

Answer: I = $\int_0^1\int_0^{1-x} g(x,y)dydx$; x varies from 0 to 1 – y ; y varies from 0 to 1.

I = $\int_0^1\int_0^{1-x} g(x,y)dydx = \int_0^1\int_0^{1-x} g(x,y)dydx$.

9. Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xydydx$.

Answer: In I_1 , x varies from 0 to $\sqrt{y}$; y varies from 0 to 1.

In I_2, x varies from 0 to 2 – y ; y varies from 1 to 2. Hence I = $I_1 + I_2$

$$I = \int_0^1 \int_{x^2}^{2-x} xydydx = \int_0^1 \int_0^{\sqrt{y}} xydxdy + \int_1^2 \int_0^{2-y} xydxdy$$

10. Change the order of integration in $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy$.

Answer: I = $\int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy$; y varies from 0 to $\frac{b}{a}\sqrt{a^2-x^2}$; x varies from 0 to a.

$$I = \int_0^b \int_0^{\frac{a}{b}\sqrt{b^2-y^2}} xydxdy = \int_0^a \int_0^{\frac{b}{a}\sqrt{a^2-x^2}} xydxdy.$$

11. Evaluate by changing the order of integration in $\int_a^0 \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xydxdy$.

Answer: I = $\int_a^0 \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xydxdy$. As per given problem strip is parallel to x axis. On changing the order of integration:

$$I = \int_a^0 \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} xydxdy = \int_0^{2a} \int_0^{\sqrt{a^2-(x-a)^2}} xydydx = \frac{2a^4}{3}.$$

12. Evaluate $\int_0^2 \int_0^3 \int_0^1 xy^2 zdzdydx$. Answer: I = $\int_0^2 \int_0^3 \int_0^1 xy^2 zdzdydx = \frac{1}{2}\int_0^2 \int_0^3 xy^2 dydx = \frac{9}{2}\int_0^2 xdx = 9.$

13. Evaluate: $\int_1^a \int_1^b \frac{dxdy}{xy}$ Answer: I = $\int_1^a \int_1^b \frac{dxdy}{xy}$ $= \int_1^a \frac{dx}{x} \int_1^b \frac{dy}{y}$ $= \log b \int_1^a \frac{dx}{x}$ $= \log b \log a.$

14. Evaluate: $\int_{-1}^2 \int_x^{x+2} dxdy$. Answer: I = $\int_{-1}^2 \int_x^{x+2} dxdy$ $= \int_{-1}^2 (y)_x^{x+2} dx$ $= 2\int_{-1}^2 dx$ = 6.

15. Evaluate: $\int_0^a \int_0^{\sqrt{a2\_x2}} dxdy$. Ans: I = $\int_0^a \sqrt{a^2-x^2}\,dx$ = $\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^a$ = $\frac{\pi a^2}{4}$.

16. Evaluate: $\int_0^a \int_0^{\sqrt{a2\_x2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$ Answer: I = $\int_0^a \left\{\sin^{-1}\left[\frac{y}{\sqrt{a^2-x^2}}\right]\right\}_0^{\sqrt{a^2-x^2}} dx$ $= \frac{\pi}{2}\int_0^a dx$ = $\frac{\pi a}{2}$.

17. Evaluate: $\int_0^4 \int_0^{\frac{4-x}{2}} zdydx$ where z = $\frac{4-x-2y}{3}$.

Answer: I $= \int_0^4 \int_0^{\frac{4-x}{2}} zdydx$ = $\int_0^4 \int_0^{\frac{4-x}{2}} \frac{4-x-2y}{3} dydx$ = $\frac{1}{3}\int_0^4 \left(\frac{4-x}{2}\right)\left[4-x-\left(\frac{4-x}{2}\right)\right]dx$ = $\frac{16}{9}$.

18. Using double integration find the area enclosed by the curves y = $2x^2$ and $y^2 = 4x$.

Answer: Given, y = $2x^2$ --------- (1) , $y^2 = 4x$ --------- (2)

Sub (1) in (2) we get x = 0 , x = 1; => y = 0 , y = 2.

Therefore the point of intersection of (1) and (2) is (0 , 0) and (1 , 2).

x varies from 0 to 1 ; y varies from $2x^2$ to $2\sqrt{x}$

The required area = $\int_0^1 \int_{2x^2}^{2\sqrt{x}} dydx$ = $2\int_0^1 \left(x^{1/2} - x^2\right)dx$ = 2/3.

19. Using double integration find the area enclosed by the curves y = x and y = x^2.

Answer: Given, y = x --------- (1) , y = x^2 --------- (2)

Sub (1) in (2) we get x = 0 , x = 1; => y = 0 , y = 1.

Therefore the point of intersection of (1) and (2) is (0 , 0) and (1 , 1).

x varies from 0 to 1 ; y varies from x^2 to x

The required area = $\int_0^1 \int_{x^2}^{x} dydx$ = $\int_0^1 (x - x^2)dx$ = 1/6.

20. Fill in the blanks: The value of $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dydx$ = $\underline{\pi/2}$

21. Fill in the blanks: The value of $\int_0^1\int_0^x e^x dydx =$ <u>(e-1)/2</u>.

22. Fill in the blanks: The value of $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dzdydx =$ <u>48</u>.

23. Fill in the blanks: The volume of the torus generated by resolving the circle $x^2 + y^2 = 4$, x =3 is $\underline{24\pi^2}$.

24. Fill in the blanks: The area between the curves $y^2 = 4x$, $x^2 = 4y$ is <u>16/3 sq. units.</u>

25. Evaluate: $\int_0^1 dx\int_0^2 dy\int_0^3 xyzdz$ Answer: I $= \int_0^1 dx\int_0^2 dy\int_0^3 xyzdz = \frac{9}{2}\int_0^1\int_0^2 xydydx = 9\int_0^1 xdx = 9/2.$

26. Evaluate: $\iiint_R (x+y+z)dxdydz$, R: $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.

Answer: $I = \iiint_R (x+y+z)dxdydz = \int_0^1\int_0^1\int_0^1 (x+y+z)dxdydz = \int_0^1\int_0^1 (\frac{1}{2}+(y+z))dydz = \int_0^1 (1+z)dz = 3/2.$

PART – B

1. Change the order of integration and hence evaluate: (a) $\int_0^1\int_{x^2}^{2-x} xydydx$, (b) $\int_0^\infty\int_0^\infty ye^{-y^2/x}$, (c) $\int_0^1\int_y^{2-y} xydxdy$,

(d) $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} xydydx$, (e) $\int_0^a\int_{x^2/a}^{2a-x} xydxdy$.

2. Evaluate: (a) $\iint xy(x+y)dxdy$ over the region bounded by $x^2 = y$ & y = x, (b) $\iint_R (x^2+y^2)dxdy$ where R is the region enclosed by x = 0, y = 0 & x + y = 1.

3. Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

4. By changing to polar coordinates, find the value of the integral (a) $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2+y^2)dydx$,

(b) $\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x}{(x^2+y^2)}dxdy$,

(c) $\int_0^2\int_0^{\sqrt{2x-x^2}} (x^2+y^2)dydx$.

5. Evaluate: (a) $\int_0^1\int_0^{1-x}\int_0^{(x+y)^2} xdzdydx$, (b) $\iiint \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$.

6. Find the volume of that portion of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which lies in the first octant.

<u>UNIT IV - FOURIER SERIES</u>

PART A

1. **If f(x) = x^2 in $(-l,l)$ is expressed as a fourier series of periodicity $2l$, find the constant term of the fourier series.**
   f(x) = x^2 is even.
   $$a_0 = \frac{2}{l}\int_0^l f(x)dx = \frac{2}{l}\int_0^l x^2dx = \frac{2}{l}\left(\frac{x^3}{3}\right) = \frac{2l^2}{3}$$ Constant term = $\frac{a_0}{2} = \frac{l^2}{3}$.
2. **State dirichlets's conditions.**
   (i)f(x) is a finite single valued function.
   (ii) f(x) has finite number of discontinuities.
   (iii) f(x) has finite number of maxima and minima.

3. **If the cosine series for f(x) = xsinx ,0< x< π is given by xsinx** **=** $1-\frac{1}{2}\cos x-2\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2-1}\cos nx$, **show that** $1+2\left[\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-...\right]=\frac{\pi}{2}$**.**

   Put x = $\frac{\pi}{2}$, which is a point of continuity.

   f(x)= f($\frac{\pi}{2}$) = $\frac{\pi}{2}$sin($\frac{\pi}{2}$) = $\frac{\pi}{2}$.   Sub x = $\frac{\pi}{2}$ in the fourier series.

   $$1-\frac{1}{2}\cos\frac{\pi}{2}-2\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2-1}\cos n\frac{\pi}{2}=\frac{\pi}{2}.$$

   $$1+2\left[\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-...\right]=\frac{\pi}{2}.$$

4. **To which value , the half range sine series corresponding to f(x) = x^2 expressed in the interval (0,2) converges at x = 2?**

   X = 2 is the end point of the range.

   f(x) = $\frac{f(0)+f(2)}{2}=\frac{0+4}{2}=2.$

5. **Find the constant term in the Fourier series corresponding to f (x) = $\cos^2 x$ expressed in the interval (-π ,π).**

   $$a_0=\frac{2}{\pi}\int_0^{\pi}\cos^2 x dx=\frac{2}{\pi}\int_0^{\pi}\frac{1+\cos 2x}{2}dx=\frac{1}{\pi}(x+\frac{\sin 2x}{2})_0^{\pi}=1.$$

   Constant term = $\frac{a_0}{2}$ = 1.

6. **Does f(x) = tan x possess a Fourier expansion?**

   f(x) = tanx has infinite discontinuities.

   It does not satisfies the dirichlets's conditions.

   Hence the fourier series does not exists.

7. **State the parseval's identity of fourier sine series in(0,π).**

   $$\bar{y}^2=\frac{1}{2}\sum_{n=1}^{\infty}b_n{}^2, \bar{y}^2=\frac{1}{\pi}\int_0^{\pi}f(x)^2dx.$$

8. **Find the value of a_n in the cosine series expansion of f (x) = k in the interval (0,10).**

   $$a_n=\frac{2}{10}\int_0^{10}k\cos\frac{n\pi x}{10}dx=\frac{2}{10}k\left(\frac{\sin\frac{n\pi x}{10}}{\frac{n\pi}{10}}\right)_0^{10}=0.$$

9. **If the Fourier series corresponding to f(x) = x in (0 , 2π) is (a_0/2)+ Σ (a_n cosnx + $b_n$sin nx) without finding the values of a_0 , a_n, b_n,   find the value of (a_0^2 / 2) + Σ (a_n^2+ b_n^2)**

   By Parseval's identity of fourier series,

   (a_0^2 / 2) + $\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ = $\frac{1}{2}\bar{y}^2$

   $$=\frac{1}{2}\left[\frac{1}{2\pi}\int_0^{2\pi}x^2dx\right]=\frac{2\pi^2}{3}.$$

10. **If f (x) is discontinuous at x = a what value does its Fourier series represent at that point.**

f(x) = $\frac{f(a+0)+f(a-0)}{2}$.

11. **What is the constant term a_0 and the coefficient of cosnx , a_n in the Fourier series expansion of f(x) = x – x³ in (-π ,π).**

f(x) = x – x^3 is an odd function. Hence, $a_0 = a_n = 0$.

12. **Choose the best answer:**

The sum of the fourier series for $f(x)=\begin{cases} x, 0<x<1 \\ 2, 1<x<2 \end{cases}$ **at x = 1 is**

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$ **(e)** $\frac{3}{2}$

Answer: (e) $\frac{3}{2}$

Because x = 1 is a point of discontinuity.

$$f(x)=\frac{f(1+0)+f(1-0)}{2}.$$

$$f(1+0)=\underset{h\to 0}{lt}\, f(1+h)=2$$

$$f(1-0)=\underset{h\to 0}{lt}\, f(1-h)=1$$

Hence f(x) = $\frac{3}{2}$.

13. **The fourier series for f(x) = x² in -1<x<1 will contain cosine terms only.**

14. **f(x)=** $\begin{cases} 1-\frac{2x}{\pi}, -\pi<x<0 \\ 1+\frac{2x}{\pi}, 0<x<\pi \end{cases}$ **is even.**

$$f(-x)=\begin{cases} 1+\frac{2x}{\pi}, -\pi<-x<0 \\ 1-\frac{2x}{\pi}, 0<-x<\pi \end{cases}$$

$$=\begin{cases} 1-\frac{2x}{\pi}, -\pi<x<0 \\ 1+\frac{2x}{\pi}, 0<x<\pi \end{cases}$$

= f(x).

15. **Using the RMS value fill up the blanks interms of fourier coefficients.**

$$\int_c^{c+2\pi}(f(x))^2dx = \underline{2\pi\left[\frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}a_n^2+b_n^2\right]}.$$

16. **Euler's formula for the fourier coefficients in the half range sine series of f(x) in (0, 2 l) are** $\underline{b_n=\frac{1}{2l}\int_0^{2l}f(x)\sin\frac{n\pi x}{l}dx}$ **.**

17. **If a periodic function f(x) is even , its fourier expansion contains only** <u>cosine terms</u>.
18. **State the convergence condition on fourier series**.
    - **i.** The fourier series of f(x) converges to f(x) at all points where f(x) is continuous.
    - **ii.** At a point of discontinuity x_0, $f(x)=\dfrac{f(x_0+0)+f(x_0-0)}{2}$
19. **Find the R.M.S value of f(x) = x in 0 < x < l.**

$$\bar{y}^2=\frac{1}{l}\int_0^l x^2dx=\frac{l^2}{3}.$$

$$\bar{y}=\frac{l}{\sqrt{3}}.$$

20. **Find the fourier sine series for the function f(x) = 1 ; 0 < x< π** .

The fourier sine series is f(x) = $\sum_{n=1}^{\infty} b_n \sin nx$ **.**

$$b_n=\frac{2}{\pi}\int_0^{\pi}\sin nx dx=\frac{2}{\pi}\left[\frac{1-(-1)^n}{n}\right].$$

$$f(x)=\sum_{n=1}^{\infty}\frac{2}{\pi}\left[\frac{1-(-1)^n}{n}\right]\sin nx$$

21. **Find the sum of the fourier series of f(x) = x+x² in -** π < x < π at x = π .

x = π is the point of discontinuity and end point of the range.

$$\therefore f(x)=\frac{f(-\pi)+f(\pi)}{2}=\pi^2.$$

22. **What do you mean by harmonic analysis?**

The process of finding the fourier series for a function y = f(x) from the tabulated values of x and y at equal intervals of x is called Harmonic analysis.

<u>PART B</u>

1. Obtain the Fourier series for f(x) of period 2l and defined as follows

f(x) = L – x , 0 < x ≤ L

= 0 , L ≤ x ≤ 2L

Hence deduce that (i) 1 – (1 / 3) + (1 / 5) – (1 / 7)+ …. = π /4,

(ii) (1 / 1^2) + (1 / 3^2) + (1 / 5^2) +….. = π^2 /8

2. Find the Fourier series expansion of period L for the function

f(x) = x in (0 , L / 2)

= L - x in (L / 2 , L)

Hence deduce the sum of the series Σ 1 / (2n – 1) ^{4}.

3. Find the half range cosine series of f(x) = (π - x^2) in the interval (0 , π). Hence find the sum of
the series (1 / 1^4) + (1 / 2^4) + (1 / 3^4) +…..

4. Find the Fourier series as the second harmonic to represent the function given in the following data.

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

5. Expand f (x) = $x^2 - x$ as Fourier series in ($-\pi, \pi$)
6. Find the Half range cosine series given f(x) = x , $0 \le x \le 1$

   = 2 - x , $1 \le x \le 2$

7. Find the Fourier series of period 2π as far as second harmonic given

X^0	0^0	30^0	60^0	90^0	120^0	150^0	180^0	210^0	240^0	270^0	300^0	330^0	360^0
Y	2.34	3.01	3.69	4.15	3.69	2.2	0.83	0.51	0.88	1.09	1.19	1.64	2.34

8. Find the Fourier series for f(x) = | cos x | in the interval ($-\pi, \pi$)
9. Find the Fourier series for f(x) = | sin x | in the interval ($-\pi, \pi$).
10. Find the Fourier series for f(x) = x sin x in the interval ($-\pi, \pi$).
11. Find the half range sine series for f(x) = x (π - x) in the interval (0, π) and deduce that ($1/1^3$) - ($1/3^3$) + ($1/5^3$) -.....

12. Obtain the half range cosine series for f(x) = x in (0 , π)
13. Find the Fourier series of f(x) = x^2 in the interval ($-\pi, \pi$).
    Hence find ($1/1^4$)+($1/2^4$)+($1/3^4$)+.....

14. Obtain a Fourier expansion for $\sqrt{(1 - \cos x)}$ in $-\pi < x < \pi$.
15. Obtain the cosine series for f(x) = x in $0 < x < \pi$ and deduce that $\Sigma\, 1 / (2n-1)^4 = \pi^4 / 96$
14. Find the Fourier series for the function f(x) = x , $0 \le x \le 1$

    = l - x , $1 \le x \le 2$

    Hence deduce that ($1/1^2$) + ($1/3^2$) + ($1/5^2$) +..... = $\pi^2/8$

15. Find the Fourier series of f(x) = 0 , $-1 < x < 0$

    = 1 , $0 < x < 1$

16. Obtain the Sine series for f(x) = x , $0 \le x \le L/2$ and f(x) = L - x , $L/2 \le x \le L$.
17. Determine the Fourier series expansion of f(x) = x in $-\pi < x < \pi$.
18. Find the half range cosine series for x sinx in (0 , π).
19. Obtain the Fourier series of period 2π for the function f(x) = 1 in (0, π)

    = 2 in (π,2π)

    Hence find the sum of ($1/1^2$) + ($1/3^2$) + ($1/5^2$) +.....

20. Obtain the Fourier series for the function

    f(x) = πx , $0 \le x \le 1$

    = π (2 – x) , $1 \le x \le 2$

21. Obtain the Fourier series for f(x) = 1 + x + x^2 in ($-\pi, \pi$).
    Deduce that ($1/1^2$) + ($1/2^2$) + ($1/3^2$) +..... = $\pi^2/6$.

22. Obtain the constant term and the first harmonic in the Fourier series expansion for f (x) where f(x) is given in the following table.

X	0	1	2	3	4	5	6	7	8	9	10	11
F(x)	18.0	18.7	17.6	15.0	11.6	8.3	6.0	5.3	6.4	9.0	12.4	15.7

23. Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $[-\pi, \pi]$.
24. Obtain the half range cosine series for $f(x0 = (x-2)^2$ in the interval $0 < x < 2$. Deduce that $\Sigma 1/(2n-1)^2 = \pi^2/8$.
25. Find the Fourier series expansion of the periodic function f (x) of period 2l defined by $f(x) = L + x, \ -L \leq x \leq 0$

$= L - x, \ 0 \leq x \leq L$. Deduce that $\Sigma 1/(2n-1)^2 = \pi^2/8$.

26. Find the half range sine series for $x \cos x$ in $(0, \pi)$.
27. Find the half range Fourier sine series for $f(x) = x^2$ in the interval $(0, \pi)$
28. Expand $f(x) = \sin x, \ 0 \leq x \leq \pi$

$= 0, \ \pi \leq x \leq 2\pi$ as a Fourier series of periodicity 2π &

Evaluate $[1/(1.3)] + [1/(3.5)] + [1/(5.7)] + \ldots\ldots\ldots$

29. Determine the Fourier sine series for the function $f(x) = x^2$ of period 2π in the interval $(0, 2\pi)$
30. Find the Half- range cosine series for the function $f(x) = x(\pi - x)$ in $0 < x < \pi$. Deduce that $(1/1^4) + (1/2^4) + (1/3^4) + \ldots = \pi^4/90$.
31. Find the complex form of Fourier series for the function $f(x) = e^{-x}$ in $-1 < x < 1$.
32. Determine the Fourier series for the function
$f(x) = -1 + x, \ -\pi < x < 0$

$= 1 + x, \ 0 < x < \pi$. Hence deduce that $1 - (1/3) + (1/5) - \ldots = \pi/4$.

32. Obtain the Fourier series of the function $f(x) = x$ for $0 < x < \Pi$

$= 2\Pi - x$ for $\Pi < x < 2\Pi$.

33. Find the Fourier series of $f(x) = (\Pi - x)^2/4$ in $0 \leq x \leq 2\Pi$. Hence deduce that
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}.$$

34. 34. Find the Fourier series of $f(x) = x^2/2$ in $-\Pi \leq x \leq \Pi$. Hence deduce that
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{12}.$$

35. If $f(x) = x + x^2$ in $-\Pi < x < \Pi$, P.T.
$$f(x) = (\Pi/3) - 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \ldots\right] + 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \ldots\right].$$

$$\text{Deduce that} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

UNIT-V- FOURIER TRANSFORMS

PART A

1. **Write the Fourier transform pair .**

   F[f(x)] = F(s) = $\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$

   f(x) = $F^{-1}[F(s)] = \frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\infty} f(x)e^{isx}ds$

2. **State the convolution theorem for Fourier transform.**

   If F[f(x)] = F(s) and F[g(x)] = G(s) then F[f(x) * g(x)] = F (s) . G(s)

   where f*g = $\frac{1}{\sqrt{2\Pi}}\int_{-\infty}^{\infty} f(t)g(x-t)dt$

3. **If $F_c[s]$ is the Fourier cosine transform of f(x) ,then the Fourier cosine transform of f(ax) is** $\underline{\frac{1}{a}F_c\left(\frac{s}{a}\right)}$

4. **If F(s) is the Fourier transform of f(x),then the formula for the Fourier transform of f(x)cosax in terms of F is** $\underline{\frac{1}{2}[F(s+a)+F(s-a)]}$

5. **Find Fourier cosine transform of e^{-x}.**

   $F_c(e^{-x}) = \sqrt{\frac{2}{\Pi}}\int_{0}^{\infty} e^{-x}\cos sx dx = \sqrt{\frac{2}{\Pi}}\left(\frac{1}{s^2+1}\right)$

6. **Find Fourier sine transform of $\frac{1}{x}$.**

   We know that $F_S[f(x)] = \sqrt{\frac{2}{\Pi}}\int_{0}^{\infty} f(x)\sin sx dx$

   $= \sqrt{\frac{2}{\Pi}}\int_{0}^{\infty}\frac{1}{x}\sin sx dx = \sqrt{\frac{2}{\Pi}}\frac{\Pi}{2}$

   $= \frac{\Pi}{2}$

7. **Find the Fourier transform of f(x) = 1, |x| < a**
   0, |x| > a > 0

   F[f(x)] = $\frac{1}{\sqrt{2\Pi}}\int_{-a}^{a} e^{isx}dx$

   $= \sqrt{\frac{2}{\Pi}}\left(\frac{\sin sa}{s}\right)$

8. **The Fourier transform of $e^{-|x|}$ is ……………..**

   a) $\sqrt{\frac{1}{2\pi}}\left[\frac{1}{s^2+1}\right]$ b) $\sqrt{\frac{1}{\pi}}\left[\frac{1}{s^2+1}\right]$ c) $\sqrt{\frac{1}{2\pi}}\left[\frac{1}{s^2-1}\right]$ d) $\sqrt{\frac{2}{\pi}}\left[\frac{1}{s^2+1}\right]$

   Ans: F(s) = F[$e^{-|x|}$] = $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-|x|}e^{isx}dx$

   $= \sqrt{\frac{2}{\pi}}\left[\frac{1}{s^2+1}\right]$

9. **The Fourier sine transform of xe^{-ax} is $\sqrt{\frac{2}{\pi}}\left[\frac{1}{(s^2+a^2)}\right]$.**

i)Say true or false
ii)Justify the statement.

False, since $F_s[xe^{-ax}] = -\frac{d}{ds}F_c[e^{-ax}] = \sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)^2}$

10. **The Fourier transform of $F[x^nf(x)]$ is ………… if the Fourier transform of f(x) is F(S).**

Ans: $(-i)^n\frac{d^n}{ds^n}F(s)$

11. **Find the Fourier transform of f(x) = 1 in (0,l).**

$F[f(x)] = \int_0^{\infty} f(x)\sin\frac{n\pi x}{l}dx = \frac{l}{n\pi}\left[1 + (-1)^n\right]$

12. **State Parseval's identity of Fourier transform.**

$\int_{-\infty}^{\infty}|f(x)|^2\,dx = \int_{-\infty}^{\infty}|f(s)|^2\,ds$

13. **Write any two applications of Fourier transforms.**

i) Solution of ODE ii) Solution of PDE

14. **$F_S\left[\frac{e^{-ax}}{x}\right]$ = …………**

a) $\sqrt{\frac{2}{\pi}}\frac{1}{s^2+a^2}$ b) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ c) $\sqrt{\frac{2}{\pi}}\tan^{-1}\left(\frac{s}{a}\right)$ d) $\sqrt{\frac{2}{\pi}}\sin^{-1}\left(\frac{s}{a}\right)$

Ans: c) since $F_S\left[\frac{e^{-ax}}{x}\right] = \sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{e^{-ax}}{x}$ sinsx dx

Differentiating on both sides w.r.t 's' we get

$$\frac{d}{ds}[F_s(s)] = \sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{e^{-ax}}{x}\frac{\partial}{\partial s}(\sin sx)\,dx$$

$$= \sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$$

Integrating on both sides w.r.t 's' we get

$$[F_s(s)] = \sqrt{\frac{2}{\pi}}\tan^{-1}\left(\frac{s}{a}\right)$$

PART B

1. Find the Fourier transform of $f(x) = \exp(-a^2x^2)$, a>0. Hence show that $\exp(-x^2/2)$ is self reciprocal under Fourier transform.

2. Find the Fourier transform of f(x) = 1 for |x|<1

= 0 otherwise.

Hence prove that $\int_0^{\infty}\sin x/x\,dx = \int_0^{\infty}\sin^2 x/x^2\,dx = \pi/2$.

3. Find the Fourier sine transform of f(x) = sinx $0<x\le\pi$

$= 0 \qquad \pi \leq x < \infty.$

4. Find the Fourier cosine transform of e^{-4x}. Deduce that

$\int_0^\infty \cos 2x \,/\, (x^2 + 16)\, dx = (\pi/8)e^{-8}$ and

$\int_0^\infty x \sin 2x \,/\, (x^2 + 16)\, dx = (\pi/2)e^{-8}$.

5. State and Prove Convolution Theorem for Fourier transform.

6. Find Fourier sine and cosine transform of e^{-x} and hence find Fourier sine transform of $x/(1+x^2)$

and Fourier cosine transform of $1/(1+x^2)$.

7. Find the Fourier transform of $e^{-a|x|}$ if $a>0$. Deduce that

(i) $\int_0^\infty 1/\, (x^2 + a^2)^2\, dx = \pi/4a^3$. and $\int_0^\infty \cos xt/\, (t^2 + a^2)\, dt = (\Pi/2a)e^{-|x|}$

8. Find the Fourier sine transform of $x.\exp(-x^2/2)$.

9. Find Fourier cosine transform of $f(x) = 1-x^2, \quad |x|\leq 1$

$= 0, \quad |x|>1.$

Hence prove that (a) $\int_0^\infty (\sin x - x\cos x) \cos(x/2) \,/\, x^3\, dx = 3\pi/16$.

(b) $\int_0^\infty [(\sin x - x\cos x) \,/\, x^3]^2\, dx = \pi/15$.

(c) $\int_0^\infty [(\sin x - x\cos x) \,/\, x^3]\, dx = \pi/4$.

10. Derive the Parseval's identity for Fourier transforms.

11. Find the Fourier sine and cosine transform of

$f(x) = \{x \quad \text{for } 0<x<1$

$2-x \quad \text{for } 1<x<2$

$0 \quad \text{for } x>2\}.$

12. Verify Parseval's Theorem of Fourier transform for the function

$f(x) = \{0 \quad \text{for } x<0$

$e^{-x} \quad \text{for } x>0\}.$

13. Find the Fourier cosine transform of $\exp(-x^2)$.

14. Find the Fourier transform of $f(x) = 1-|x|$ if $|x|<1$.

$= 0 \quad |x|\geq 1$. Hence deduce $\int_0^\infty (\sin t/t)^4\, dt = \pi/3$.

15. Find the Fourier sine transform of e^{-ax}/x, where $a>0$.

16. Evaluate $\int_0^\infty 1/\, (x^2 + a^2)(x^2 + b^2)\, dx$ using Fourier transforms.

17. Find the Fourier cosine transform of $\exp(-a^2x^2)$. Hence find the Fourier sine transform of $x \exp(-a^2x^2)$.

18. Find the Fourier transform of $\exp(-a^2x^2)$. Hence prove $\exp(-x^2/2)$ is self reciprocal.

19. Find Fourier sine and cosine transform of x^{n-1} where $0<n<1$, $x>0$. Deduce that $1/\sqrt{x}$ is self reciprocal under both Fourier sine and cosine transform.

20. Using Parseval's identity for Fourier cosine transform of e^{-ax} & Hence find $\int_0^\infty 1/(x^2 + a^2)^2\, dx$.

21. Find the Fourier sine transform of e^{-ax} $(a>0)$. Hence find $F_s\{xe^{-ax}\}$.

22. Find the Fourier sine transform of $f(x) = \sin x,\ 0<x<a$

$$= 0,\quad x>a.$$

23. If $F[f(x)] = F(s)$, prove that $F[f(ax)] = (1/|a|)F(s/a)$.

24. Find the Fourier transform of $e^{-a|x|}$, $a>0$. Hence deduce that $F[x\, e^{-a|x|}] = i[2as/(a^2+s^2)^2]\sqrt{(2/\pi)}$.

25. Find the Fourier sine transform of the function $f(x) = e^{-ax}/x$.

26. Find the Fourier transform of the function $e^{-|x|}$. Using Parseval's identity $\int_0^\infty dx/(x^2+1)^2 = \Pi/4$

27. Prove that the Fourier tranform of $\exp(-x^2/2)$ is $\exp(-s^2/2)$ & deduce that $F(x.\exp(-x^2/2)) = (is) \exp(-x^2/2)$.

28. Find the Fourier cosine transform of $f(x) = \{\cos x \quad 0<x\leq \pi$

$$0 \qquad \pi\leq x<\infty\}.$$

29. Find the Fourier sine transform of $e^{-|x|}$.

Hence show that $\int_0^\infty (x\sin mx)/(1+ x^3)dx = \Pi\, e^{-m}/2$, $m>0$.

30. Find the Fourier cosine transform of e^{-4x}.

Deduce that $\int_0^\infty (\cos 2x)/(16+ x^2)dx = \Pi\, e^{-8}/8$, and

$\int_0^\infty (x\sin 2x)/(16+ x^2)dx = \Pi\, e^{-8}/2$.

****** ***ALL THE BEST*** ******

K.S.R COLLEGE OF ENGINEERING (Autonomous)

Vision of the Institution

- We envision to achieve status as an excellent educational institution in the global knowledge hub, making self-learners, experts, ethical and responsible engineers, technologists, scientists, managers, administrators and entrepreneurs who will significantly contribute to research and environment friendly sustainable growth of the nation and the world.

Mission of the Institution

- To inculcate in the students self-learning abilities that enable them to become competitive and considerate engineers, technologists, scientists, managers, administrators and entrepreneurs by diligently imparting the best of education, nurturing environmental and social needs.
- To foster and maintain mutually beneficial partnership with global industries and institutions through knowledge sharing, collaborative research and innovation.

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

Vision of the Department

- We envision a department that leads in the field of Electrical and Electronics Engineering through education, training and research committed to influence the direction of the field and make constructive contribution to society wherein the department can thrive and grow.

Mission of the Department

- To create professionally competent and resourceful Electrical and Electronics Engineers.
- To promote excellence in teaching, pioneering research and innovation for a sustainable growth of the nation and enrichment of humanity.

Programme Educational Objectives (PEOs):

- Excel in professional career and /or higher education by acquiring knowledge in basic engineering, science and mathematics in Electrical and Electronics Engineering.
- Develop and apply engineering solutions for solving contemporary social and human issues with realistic constraints through modern tools.
- Exhibit professional and ethical standards, effective communication skills, teamwork spirit and multidisciplinary approach for successful careers in Indian and Multinational companies and to engage in lifelong learning.