K.S.R COLLEGE OF ENGINEERING (AUTONOMOUS) - THIRUCHENGODE
DEPARTMENT OF MATHEMATICS
QUESTION BANK
18MA151 – ENGINEERING MATHEMATICS - I

UNIT - I LINEAR ALGEBRA
PART A (QUESTIONS UNDER CO1)

1. Construct the characteristic equation of $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (Creating)

$S_1 = 5$, $S_2 = -6$, The characteristic equation is $\lambda^2 - 5\lambda - 6 = 0$

2. The product of two eigen values of the matrix A = $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value. (Remembering)

Let the eigen values of the matrix A is λ_1, λ_2, λ_3. Given that $\lambda_1 \lambda_2 = 16$. We know that $\lambda_1 \lambda_2 \lambda_3$ = determinant of A = 32. (verify yourself) i.e. $\lambda_1 \lambda_2 \lambda_3 = 32 \Rightarrow 16 \lambda_3 = 32 \Rightarrow \lambda_3 = 2$.

3. Two eigen values of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ are 0 and 1, obtain the third eigen value. (Remembering)

Given that $\lambda_1 = 0$, $\lambda_2 = 1$ then find λ_3 = ?. W.K.T. sum of the eigen values = sum of the main diagonal elements. i.e. $\lambda_1 + \lambda_2 + \lambda_3 = 11 + (-2) + (-6) = 3$.
i.e. $0 + 1 + \lambda_3 = 3 \Rightarrow \lambda_3 = 2$. Hence the third eigen value = 2.

4. Two eigen values of the matrix A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. What is the third eigen value?. Also find the product of the eigen values of A. (Remembering)

Given that $\lambda_1 = 3$, $\lambda_2 = 0$ then find λ_3 = ?.
By property, $\lambda_1 + \lambda_2 + \lambda_3 = 8 + 7 + 3 = 18 \Rightarrow \lambda_3 = 15$.
Hence product of the eigen values = $\lambda_1 \lambda_2 \lambda_3 = (3)(0)(15) = 0$.

5. One of the eigen values of $\begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ 4 & -1 & -8 \end{bmatrix}$ is -9. Find the other two eigen values. (Remembering)

W.K.T. sum of the eigen values = sum of the diagonal elements.
i.e. $\lambda_1 + \lambda_2 + \lambda_3 = 7 + (-8) + (-8) = -9$.
Given that $\lambda_3 = -9$. Hence we get, $\lambda_1 + \lambda_2 - 9 = -9 \Rightarrow \lambda_1 + \lambda_2 = 0$ …….
(i)Product of eigen values = |A| = 441 (verify yourself).
i.e. $\lambda_1 \lambda_2 \lambda_3 = 441 \Rightarrow \lambda_1 \lambda_2 (-9) = 441 \Rightarrow \lambda_1 \lambda_2 = -49$ ….. (ii) From (i), $\lambda_2 = -\lambda_1$.
From(ii), $\lambda_1 \lambda_2 = -49 \Rightarrow \lambda_1(-\lambda_1) = -49 \Rightarrow -\lambda_1^2 = -49 \Rightarrow \lambda_1 = \pm 7$.
If $\lambda_1 = 7$ then $\lambda_2 = -7$ and if $\lambda_1 = -7$ then $\lambda_2 = 7$.
Hence the remaining two eigen values are 7 and -7.

6. The matrix A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ 1 & 2 & 3 \end{bmatrix}$ is singular. One of its eigen value is 2.

Find the other two eigen values. (Remembering)

A = $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & -2 \\ 1 & 2 & 3 \end{bmatrix}$, let the eigen values be $\lambda_1, \lambda_2, \lambda_3.$

Given $\lambda_1 = 2$. Sum of the eigen values = $\lambda_1 + \lambda_2 + \lambda_3 = 1 + 0 + 3 \Rightarrow 2 + \lambda_2 + \lambda_3 = 4$
$\Rightarrow \lambda_2 + \lambda_3 = 2$.
Product of the eigen values = det. Of A = -8 (verify). i.e. $\lambda_1 \lambda_2 \lambda_3 = -8$
$\Rightarrow 2\lambda_2\lambda_3 = -8 \Rightarrow \lambda_2\lambda_3 = -4$.
X^2 – (sum of the eigen values)x + product of the eigen values = 0.
$\Rightarrow x^2 - 2x + (-4) = 0 \Rightarrow x = 1 \pm \sqrt{5}$. Hence $\lambda_2 = 1 + \sqrt{5}$ and $\lambda_2 = 1 - \sqrt{5}$.

7. Form the matrix whose eigen values are α-5, β-5, γ -5 where α,β, γ are the eigen values of

A = $\begin{bmatrix} -1 & -2 & -3 \\ 4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix}$. (Understanding)

W.K.T the matrix A-KI has the eigen values λ_1-k, λ_2-k….. λ_n-k.

Hence the matrix is A-5I = $\begin{bmatrix} -1 & -2 & -3 \\ 4 & 5 & -6 \\ 7 & -8 & 9 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -6 & -2 & -3 \\ 4 & 0 & -6 \\ 7 & -8 & 4 \end{bmatrix}$.

8. Find the constants ‘a’ and ‘b’ such that the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as its eigen values. (Remembering)

W.K.T. sum of the eigen values of a matrix = sum of the elements of the main diagonal.
$\Rightarrow (3) + (-2) = a + b \Rightarrow a + b = 1$ …
(i)Product of the eigen values = determinant of the matrix

i.e. (3) (-2) = $\begin{vmatrix} a & 4 \\ 1 & b \end{vmatrix} \Rightarrow -6 = ab - 4 \qquad \Rightarrow ab = -2.$ …

(ii)Solving (i) and (ii) we get $x^2 - (a+b)x + ab = 0 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, -1$.
Therefore a = 2, b = -1 or a = -1, b = 2.

9. If 1,1,5 are the eigen values of A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ find the eigen values of 5A. (Remembering)

If λ_1, λ_2, λ_3 be the eigen values of A then kλ_1, kλ_2, kλ_3 be the eigen values of KA.
Therefore the eigen values of 5A are 5, 5, 25.

10. If 2,3 are the eigen values of $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ a & 0 & 2 \end{bmatrix}$. Find the value of a. (Remembering)

Let λ_1, λ_2, λ_3 be the eigen values of A. Given $\lambda_1 = 2$, $\lambda_2 = 3$.

W.K.T. $\lambda_1 + \lambda_2 + \lambda_3$ = sum of the main diagonal elements $\Rightarrow 2 + 3 + \lambda_3 = 2 + 2 + 2 \Rightarrow 5 + \lambda_3 = 6 \Rightarrow \lambda_3 = 1$.

Also W.K.T. $\lambda_1 \lambda_2 \lambda_3 = |A| \Rightarrow (2)(3)(1) = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ a & 0 & 2 \end{vmatrix}$

$\Rightarrow 6 = 8 - 2a \Rightarrow a = 1$.

11.Obtain the Eigen values of adj A if A = $\begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 2 \\ 0 & 0 & 1 \end{bmatrix}$. (Understanding)

Since A is a upper triangular matrix, the eigen values of A are 3 , 4 , 1.

W.K.T. $A^{-1} = \frac{1}{|A|} \text{adj}A \Rightarrow \text{adj } A = |A|\, A^{-1}$.

The eigen values of A^{-1} are1/3 , ¼ , 1. |A| = Product of the eigen values = 12.
Therefore the eigen values of adjA is equal to the eigen values of $12A^{-1}$.

i.e. $\frac{12}{3}, \frac{12}{4}$, 12. i.e.4,3,12.

Hence the eigen values of adj A = 4, 3, 12.

12.Two eigen values of A = $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ are equal they are double the third. Find the eigen values of A^2.

(Remembering)

Let the third eigen values be λ. The remaining two eigen values are 2λ, 2λ.

Sum of the eigen values = sum of the main diagonal elements $\Rightarrow 2\lambda + 2\lambda + \lambda = 4 + 3 + (-2) \Rightarrow \lambda = 1$.
Therefore the eigen values of A are 2, 2, 1. Hence the eigen values of A^2 are 2^2, 2^2, 1^2. i.e. 4, 4, 1.

13. If A = $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, then develop the eigen values of $3A^3 + 5A^2 - 6A + 2I$. (Creating)

Since A is an upper triangular matrix, the eigen values of A are 1, 3, -2.

Let $\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = -2$.

Let the eigen values of $3A^3 + 5A^2 - 6A + 2I$ are k_1, k_2, k_3.

$\lambda_1 = 1 \Rightarrow k_1 = 3(1)^3 + 5(1)^2 - 6(1) + 2 = 3 + 5 - 6 + 2 = 4$.

$\lambda_2 = 3 \Rightarrow k_2 = 3(3)^3 + 5(3)^2 - 6(3) + 2 = 3(27) + 5(9) - 18 + 2 = 110$.

$\lambda_3 = -2 \Rightarrow k_3 = 3(-2)^3 + 5(-2)^2 - 6(-2) + 2 = 3(-8) + 5(4) + 12 + 2 = 10$.

Therefore the required eigen values are 4, 110, 10.

14. Prove that eigen values of $-3A^{-1}$ are the same as those of A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. (Evaluating)

The characteristic equation of A is $|A - \lambda I| = 0 \Rightarrow \lambda^2 - 2\lambda - 3 = 0 \Rightarrow \lambda = 3, -1$.

Hence the eigen values of A = -1, 3. … (i) Therefore the eigen values of A^{-1} are -1, 1/3.

The eigen values of $-3A^{-1}$= (-3)(-1) , (-3) (1/3). i.e. 3, -1….

(ii)From (i) and (ii), we conclude that the eigen values of ($-3A^{-1}$) are same as those of A.

15.The eigen values of the matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ are distinct. If the eigen vectors of the given matrix are $\begin{bmatrix} 1 \\ a \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} b \\ -2 \\ 1 \end{bmatrix}$.
Find the value of a & b. (Remembering)

Given matrix is a symmetric matrix. Hence the eigen vectors are orthogonal.

Let $X_1 = \begin{bmatrix} 1 \\ a \\ 2 \end{bmatrix}$, $X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $X_3 = \begin{bmatrix} b \\ -2 \\ 1 \end{bmatrix}$.

Now $X_1^T X_2 = 0 \Rightarrow \begin{bmatrix} 1 & a & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} = 2 + a - 4 = 0 \Rightarrow a = 2$.

Similarly $X_2^T X_3 = 0 \Rightarrow \begin{bmatrix} 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} b \\ -2 \\ 1 \end{bmatrix} = 2b - 2 - 2 = 0 \Rightarrow b = 2$.

16. Explain Cayley-Hamilton theorem. (Understanding)
Every square matrix satisfies its own characteristic equation.

17.Explain the uses of Cayley-Hamilton theorem. (Understanding)
To calculate (i) the positive integral powers and
(ii) the inverse of a given square matrix.

18.Show that the matrix $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ satisfies its own characteristic equation. (Or)

Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ (Understanding)

Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$. The cha. Eqn. of A is $|A - \lambda I| = 0 \Rightarrow \lambda^2 - S_1\lambda + S_2 = 0$.

S_1 = sum of the main diagonal elements = 1 + 1 = 2. $S_2 = |A| = 5$ (verify).
Hence the cha. Eqn. becomes $\lambda^2 - 2\lambda + 5 = 0$. To prove that $A^2 - 2A + 5I = 0$.

$A^2 = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix}$.

$\therefore A^2 - 2A + 5I = \begin{bmatrix} -3 & -4 \\ 4 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Therefore the given matrix satisfies its own characteristic equation.

19.Using Cayley Hamilton theorem, find the inverse of $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. (Applying)

Given $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. The char eqn. of A is $|A - \lambda I| = 0 \Rightarrow A^2 - 4A - 5I = 0$ ….(i) (verify yourself)

Divide (i) by A we get $A - 4I = \frac{5}{A} \Rightarrow 5A^{-1} = A - 4I$

$\Rightarrow A^{-1} = \frac{1}{5}[A - 4I] = \frac{1}{5}\left[\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]$. Therefore $A^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 2 & -1 \end{bmatrix}$.

20.Use Cayley Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ to express

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 101$ as a linear polynomial in A. (Applying)

Given: A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. The cha. Eqn. of A is $|A - \lambda I| = 0 \Rightarrow \lambda^2 - 4\lambda - 5 = 0$ (verify yourself).

$\therefore$ By Cayley Hamilton theorem, we have $A^2 - 4A - 5I = 0$. …(i)

$A^5 - 4A^4 - 7A^3 + 11A^2 - A - 101 = (A^2 - 4A - 5I)(A^3 - 2A + 3) + A + 5 = 0 + A + 5$

by (i) which is a linear polynomial in A.

21.Define the quadratic form. (Remembering)

A homogeneous polynomial of the second degree in any number of variables is called a quadratic form.

22.Write the matrix form of the Quadratic form $2x^2 + 8z^2 + 4xy - 10xz - 2yz$. (Remembering)

$$\text{Quadratic form} = \begin{bmatrix} coeff.of.x^2 & \frac{1}{2}coeff.of.xy & \frac{1}{2}coeff.of.xz \\ \frac{1}{2}coeff.of.xy & coeff.of.y^2 & \frac{1}{2}coeff.of.yz \\ \frac{1}{2}coeff.of.xz & \frac{1}{2}coeff.of.yz & coeff.of.z^2 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 5 \\ 2 & 0 & -1 \\ 5 & -1 & 8 \end{bmatrix}.$$

23.Find the nature of the quadratic form $6x^2 + 3y^2 + 14z^2 + 4yz + 18xz + 4xy$ without finding the eigen values. (Remembering)

$D_1 = |6| = 6$ (+ve), $D_2 = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix} = 14$ (+ve), $D_3 = \begin{bmatrix} 6 & 2 & 9 \\ 2 & 3 & 2 \\ 9 & 2 & 14 \end{bmatrix} = 1$ (+ve)

Since, D_1, D_2, and D_3 are positive, the quadratic form is positive definite.

24.Determine the nature of the following quadratic form. $F(x_1,x_2,x_3) = x_1^2 + 2x_2^2$. (Evaluating)

The matrix of the quadratic form is Q = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ by using the formula in the previous question.

$D_1 = |1| = 1$ (+ve), $D_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = 2$ (+ve), $D_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$ (verify).

Therefore the quadratic form is positive definite.

25. When do you say a quadratic form is positive definite? (Remembering)

A quadratic form is said to be positive definite if all the eigen values are positive.

26. Write the equivalent quadratic form of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Given matrix form is $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$. (Remembering)

Therefore its equivalent quadratic form is $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$.

PART B (QUESTIONS UNDER CO1)

1. Find the Eigen values and Eigen vectors of the following matrices: (Applying)

(a) $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$

(d) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ (e) $\begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$ (f) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.

2. Using Cayley – Hamilton theorem, find A^4 & A^{-1} when

(a) $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$ (c) $A = \begin{pmatrix} -1 & 0 & 3 \\ 8 & 1 & -7 \\ -3 & 0 & 8 \end{pmatrix}$. (Applying)

3. Verify Cayley – Hamilton theorem & hence find $^{A-1}$ for the following matrices:

(a) $\begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. (Understanding)

4. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then show that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$ using Calyey – Hamilton Theorem. (Applying)

5. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$, find A^n in-terms of A. (Remembering)

6. Reduce the quadratic form into canonical form by an orthogonal transformation and hence Determine its nature:

(a) $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4zx$, (b) $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$,

(c) $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$, (d) $x_1^2 + 2x_2x_3$. (e) $2xy+2yz+2zx$ (Remembering)

UNIT - II
ORDINARY DIFFERENTIAL EQUATIONS
PART A (QUESTIONS UNDER CO2)

1. Define linear differential equation of second order. (Remembering)

Solution: Linear differential equation of second order with constant coefficient is defined as,

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = X \Rightarrow (D^2 + a_1 D + a_2)y = X.$$

2. Find P.I of $(D^2 - 4D + 13)y = e^{2x}$. (Understanding)

Solution: P.I = $\dfrac{1}{D^2 - 4D + 13}e^{2x} \Rightarrow \dfrac{1}{4 - 8 + 13}e^{2x} \Rightarrow \dfrac{1}{9}e^{2x}$.

3. Give the particular integral for $(D^2 + 4)y = \sin 2x$. (Applying)

Solution: P.I= $\dfrac{\sin 2x}{D^2 + 2^2} = -\dfrac{x\cos\alpha x}{2\alpha} = -x\dfrac{\cos 2x}{4}$.

4. Solve: $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0.$ (Understanding)

Solution:

$(D^2+6D+9)y=0\Rightarrow$ $A..E: m^2+6m+9=0\Rightarrow(m+3)(m+3)=0$ $\Rightarrow m=-3,-3.$

Roots are real and equal. $\therefore$ $y=(C_1+C_2x)e^{-3x}.$

5. Find the C.F for $(D^2-2D+4)y=e^x\sin 2x.$ (Applying)

Solution: $(D^2-2D+4)=0\Rightarrow$ $m^2-2m+4=0\Rightarrow m=1\pm i\sqrt{3}$

C.F = $e^x\left[A\cos\sqrt{3}x+B\sin\sqrt{3}x\right]$

6. Reduce the equation $(2x+3)^2y''-(2x+3)y'-12y=6x$ into a linear differential equation with constant coefficient. (Understanding)

Solution: This is the Legendre's linear equation: $\left((2x+3)^2D^2-(2x+3)D-12\right)y=6x$ … (1)

Put $z=\log(2x+3),\ e^z=2x+3\Rightarrow(2x+3)D=2\theta\Rightarrow(2x+3)^2D^2=4(\theta^2-\theta),\theta=\frac{d}{dz}$

Put in (1): $(4\theta^2-6\theta-12)y=3e^z-9.$

7. Find the particular integral of $(D-1)^2y=\sinh 2x.$ (Understanding)

Solution: $P.I=\frac{1}{(D-1)^2}\left(\frac{e^{2x}-e^{-2x}}{2}\right)=\frac{1}{2}\left[\frac{e^{2x}}{(2-1)^2}-\frac{e^{-2x}}{(-2-1)^2}\right]\Rightarrow\frac{e^{2x}}{2}-\frac{e^{-2x}}{18}.$

8. Solve $\frac{d^2x}{dt^2}+n^2x=0.$

$A.E=m^2+n^2=0\Rightarrow m=\pm in\Rightarrow x=A\cos nt+B\sin nt.$

9. Solve $(x^2D^2-3xD)y=0.$ (Applying)

Solution: Choose $x=e^z, z=\log x$

$(\theta^2-4\theta)y=0\Rightarrow m^2-4m=0\Rightarrow m=0,m=4.\Rightarrow y=Ae^{0z}+Be^{4z}=A+Be^{4z}.$

10. Solve $\frac{d^2y}{dx^2}=y.$ (Understanding)

Solution: $(D^2-1)y=0\Rightarrow A.E: m^2-1=0\Rightarrow m=\pm1\Rightarrow y=Ae^{-x}+Be^{x}.$

11. Transform the equation $(2x-1)^2y''-4(2x-1)y'+8y=8x$ into the linear differential equation with constant coefficient.

Solution: $2x-1=e^z(or)z=\log(2x-1)(2x-1)D=2\theta,(2x-1)^2D^2=4\theta(\theta-1)$

Put in (1): $(\theta^2-3\theta+2)y=\frac{8}{4}\left(\frac{e^z+1}{2}\right)=(e^z+1)\Rightarrow(\theta^2-3\theta+2)y=(e^z+1).$

12. Find the particular integral of $(D^2-2D+4)y=e^x\cos x.$ (Applying)

Solution: $P.I=\frac{e^x\cos x}{(D^2-2D+4)}=e^x\frac{1}{(D+1)^2-2D(+1)+4}(\cos x)\Rightarrow e^x\frac{1}{D^2+3}\cos x=\frac{e^x\cos x}{2}.$

13. Solve $(D^2+1)^2 y = 0.$

Solution: $A.E = (m^2+1)(m^2+1) = 0 \Rightarrow m = \pm i, m = \pm i. \Rightarrow y = (C_1 + C_2 x)\cos x + (C_3 + C_4 x)\sin x.$

14. Solve $x^2 \dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 2y = 0.$ (Applying)

Solution: $x = e^z, z = \log x,\ xD = \theta, x^2D^2 = \theta(\theta-1) \Rightarrow (\theta^2 + 3\theta + 2)y = ze^z$

$A.E : m^2 + 3m + 2 = 0 \Rightarrow m = -1, -2. \Rightarrow y = Ae^{-z} + Be^{-2z}.$

15. Reduce the equation $x^2 \dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = x$ in to a linear differential equation with constant coefficients.

Solution: $Put \quad z = \log x, x = e^z,\ xD = \theta, x^2D^2 = \theta(\theta-1) \Rightarrow (\theta^2 - 2\theta + 1)y = e^z.$

16. Solve $(D^2 + D + 1)y = 0.$ (Applying)

Solution: $A.E : m^2 + m + 1 = 0 \Rightarrow m = \dfrac{-1 \pm i\sqrt{3}}{2} = -\dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2} \Rightarrow \alpha = -\dfrac{1}{2}, \beta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow C.F = e^{-\frac{x}{2}}\left[A\cos\dfrac{\sqrt{3}}{2}x + B\sin\dfrac{\sqrt{3}}{2}x\right] \Rightarrow y = e^{-\frac{x}{2}}\left[A\cos\dfrac{\sqrt{3}}{2}x + B\sin\dfrac{\sqrt{3}}{2}x\right].$

17. Write the Cauchy's homogeneous linear equation. (Remembering)

Solution: $x^n \dfrac{d^ny}{dx^n} + a_1 x^{n-1}\dfrac{d^{n-1}y}{dx^{n-1}} + ... + a_n y = X.$

18. Find the particular integral of $(D+1)^2 y = e^{-x}\cos x.$ (Remembering)

Solution: $P.I = \dfrac{1}{(D+1)^2}e^{-x}\cos x \Rightarrow e^{-x}\dfrac{1}{D^2}\cos x \Rightarrow e^{-x}(-\cos x).$

19. Solve $r\dfrac{d^2u}{dr^2} + \dfrac{du}{dr} = 0.$ (Applying)

Solution: $r^2\dfrac{d^2u}{dr^2} + r\dfrac{du}{dr} = 0\ and\ \ put: z = \log r, r = e^z, D = \dfrac{d}{dr} = \theta, D^2 = \dfrac{d^2}{dr^2} = \theta(\theta-1)$

$\theta^2 = 0 \Rightarrow u = Az + B = A\log r + B.$

20. Find the particular integral of $(D^3 - 1)y = e^{2x}.$ (Applying)

Solution: P.I = $\dfrac{e^{2x}}{D^3 - 1} = \dfrac{e^{2x}}{8-1} = \dfrac{e^{2x}}{7.}.$

21. Find the particular integral of $(D^2 + 4)y = \cos 2x.$ (Applying)

Solution: P.I = $\dfrac{\cos 2x}{D^2 + 2^2} = \dfrac{x\sin 2x}{2(2)} = \dfrac{x\sin 2x}{4}.\left[\Theta\ \dfrac{\cos\alpha x}{2\alpha} = \dfrac{x\sin\alpha x}{2\alpha}\right]$

22. Find the particular integral of $(D^2+4D+4)y=xe^{-2x}$. (Applying)

Solution: $P.I=\frac{1}{(D+2)^2}e^{-2x}(x)\Rightarrow \frac{e^{-2x}x^3}{6}$.

PARTB (QUESTIONS UNDER CO2)

1.Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$. (Applying)

2.Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=(1+x)$.

3. Solve $(D^2 -4D + 4)y = e^{2x} +\cos 2x$

4. Solve $(D^2-4D+3)y=\sin 3x\cos 2x$.

5. Solve $(D^2-4D+4)y=e^{2x}x^4+\cos 2x$.

6. Solve $(D^2+4)y=4e^{2x}\sin 3x$.

7. Solve $(D^2+4D+3)y=e^{-x}\sin x+xe^{3x}$.

8. Solve $(x^2D^2-7xD+12)y=x^2$

9. Solve $(D^2+4)y=x^4+\cos^2 x$.

10. Solve $(x^2D^2+4xD+2)y=\cos(\log x)$.

11.Solve $x^2y''+3xy'+5y=x\cos(\log x)$.

12.Solve $(x^2D^2+xD+4)y=\log x\sin(\log x)$.

13.Solve $(x^2D^2+4xD+2)y=x\log x$.

14.Solve $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x^2+\frac{1}{x^2}$.

15.Solve $(2x+3)^2y''-2(2x+3)y'-12y=6x$.

16.Solve $(D^2+9)y=\cot 3x$.

17.Solve $y''-\frac{4}{x}y'+\frac{4}{x^2}y=x^2+1$ by the method of variation of parameter.

18.Solve $\frac{d^2y}{dx^2}+a^2y=\sec ax$, by the method of variation of parameter.

19.Solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec}x\cot x$ by the method of variation of parameters.

20.Solve $\frac{d^2y}{dx^2}+4y=\tan 2x$ by the method of variation of parameter.

21.Solve by the method of variation of parameter $(D^2+16)y=\sec^2 4x$.

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UNIT - III

DIFFERENTIAL CALCULUS
PART A (QUESTIONS UNDER CO3)

1. Find the radius of curvature for the curve $y = \frac{x^3-a^3}{a^2}$ at any point (x, y) (Remembering)

Solution: Given $a^2 = x^3 - y^3$

Differentiating w.r.t. 'x' we get $a^2\frac{dy}{dx} = 3x^2$; $\frac{dy}{dx} = \frac{3x^2}{a^2}\ \frac{d^2y}{dx} = \frac{6x}{a^2}$

$$\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left\{1+\frac{9x^4}{a^4}\right\}^{\frac{3}{2}}}{\frac{6x}{a^2}}$$

2. Solve ρ for the curve y = 4 sin x – sin 2x at x = 90^{o}. (Applying)

Solution : Given y = 4 sin x – sin 2x

$\frac{dy}{dx} = 4\cos x - 2\cos 2x$ $\quad\quad$ $\frac{d^2y}{dx^2} = -4\sin x - 4\sin 2x$

$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 2$ $\quad\quad$ $\left(\frac{d^2y}{dx^2}\right)_{x=\frac{\pi}{2}} = -4$

$$\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{(1+4)^{\frac{3}{2}}}{-4} = \frac{5\sqrt{5}}{-4}$$

3. Find the envelope of the family of lines $\frac{x}{t} + yt = 2c$, t being the parameter. (Remembering)

Sol: Given $\frac{x}{t} + yt = 2c$. $x + yt^2 - 2ct = 0$ Which is quadratic in 't'. A= y, B =-2c, C= x.

Envelope is B^2 – 4 A C =0 $\Rightarrow$ $4C^2$ -4 x y=0 $\Rightarrow$ $c^2 = xy$.

4. Solve the radius of curvature at y = 2a on the curve y^2 = 4a x. (Applying)

Sol: Given y^2 = 4a x. Diff.w.r.t x. , $2y\frac{dy}{dx}$ = 4a. When y= 2a, x=$\frac{y^2}{4a}$=a. $\Rightarrow \left(\frac{dy}{dx}\right)_{(a,2a)}$ =1 ;

$\left(\frac{d^2x}{dy^2}\right)_{(a,2a)}$ = -1/2a. $\quad\quad$ $\rho = \frac{[1+(\frac{dy}{dx})^2]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ = 2a.2 $^{3/2.}$

5. Define curvature and radius of curvature. (Remembering)

Solution: Curvature = Rate of bending = $\frac{d\psi}{ds}$; Radius of curvature = Reciprocal of curvature, $\rho = \frac{ds}{d\psi}$

6. Prove the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$ is $\frac{a}{\sqrt{2}}$. (Evaluating)

Solution: Given $\sqrt{x} + \sqrt{y} = \sqrt{a}$ ………….. (1)

Diff. (1) w.r.t. 'x' we get,

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}.\frac{dy}{dx} = 0$ $\quad\quad$ $\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$

$\left(\frac{dy}{dx}\right)_{\left(\frac{a}{4},\frac{a}{4}\right)} = -1$

$$\frac{d^2y}{dx^2} = \frac{-\left[\sqrt{x}\left(\frac{1}{2\sqrt{y}}\frac{dy}{dx}\right) - \sqrt{y}\frac{1}{2\sqrt{x}}\right]}{x}$$

$$\left(\frac{d^2y}{dx^2}\right)_{\frac{a}{4},\frac{a}{4}} = \frac{\frac{1}{2}+\frac{1}{2}}{\frac{a}{4}} = \frac{4}{a} \therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{(1+1)^{\frac{3}{2}}}{\frac{4}{2}} = \frac{a}{4}.2^{\frac{3}{2}} = \frac{a.2\sqrt{2}}{4} = \frac{a}{\sqrt{2}}$$

7. Find the radius of curvature of the curve $e^{\frac{y}{a}} = \sec\left(\frac{x}{a}\right)$ at any point (Remembering)

Solution: Given $e^{\frac{y}{a}} = \sec\left(\frac{x}{a}\right)$

$$\frac{y}{a} = \log\left[\sec\left(\frac{x}{.a}\right)\right] \; y = a\log\left[sec\left(\frac{x}{.a}\right)\right]$$

$$\frac{dy}{dx} = a.\frac{1}{\sec\frac{x}{a}}.e^{\frac{y}{a}} = \sec\frac{x}{a}.tan\frac{x}{a}.\frac{1}{a} = \tan\frac{x}{a}$$

$$\frac{d^2y}{dx} = \sec 2\frac{x}{a}.\frac{1}{a} \;\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{a\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\sec 2\frac{x}{a}} = a\sec\frac{x}{a}$$

8.Determine ρ for the curve x = a cos θ, y = a sin θ at θ. (Evaluating)

Solution: Given x = a cos θ & y = a sin θ

$$\frac{dx}{d\theta} = -a\sin\theta \quad \frac{dy}{d\theta} = -a\cos\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} + \frac{a\cos\theta}{a\sin\theta} + \cot\theta$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(-\cot\theta) = cosec^2\theta \; X \; \frac{d\theta}{dx} = cosec^2\theta \; X \; \frac{1}{a\sin\theta} = \frac{1}{a\sin^3\theta}$$

$$\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{(1+\cot^2\theta)^{\frac{3}{2}}}{\frac{1}{a\sin^3\theta}} = \text{cosec}^3\,\theta \text{ x a} \sin^3\theta = \text{a}$$

9. Find the radius of curvature of the curve $\text{x} = \text{at}^2, \text{y} = \text{at}$ at any point‘t’. (Remembering)

Solution: Given $\text{x} = \text{at}^2$, y = 2at

$$\frac{\text{dy}}{\text{dx}} = 2\text{ at;} \qquad \frac{\text{dy}}{\text{dx}} = 2\text{a}$$

$$\frac{\text{dy}}{\text{dx}} = \frac{2\text{a}}{2\text{at}} = \frac{1}{\text{t}}$$

$$\frac{\text{d}^2\text{y}}{\text{dx}} = \frac{\text{d}}{\text{dx}}\left(\frac{\text{dy}}{\text{dx}}\right) = \frac{\text{d}}{\text{dx}}\left(\frac{1}{\text{t}}\right) = \frac{-1}{\text{t}^2}.\frac{\text{dt}}{\text{dx}} = \frac{-1}{\text{t}^2}.\frac{1}{2\text{at}}$$

$$\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\left(\frac{1}{t}\right)^2\right]^{\frac{3}{2}}}{\frac{-1}{2at^3}} = \frac{(t^2+1)^{\frac{3}{2}}}{t^3} \; x \; 2at^3 = 2a\,(1+t^2)^{\frac{3}{2}}$$

10. What is the curvature of a circle of radius ‘ r ’?. (Remembering)

Solution: The curvature of a circle of radius $r = \frac{1}{r}$.

Radius of curvature = 1/curvature of a curve.

11. What is the curvature of a circle $x^2 + y^2 = 25$ at any point on it.. (Remembering)

Solution: The given curve is a circle of radius 5. Therefore the curvature at any point on the circle $\frac{1}{r} = \frac{1}{5}$.

12. Estimate the radius of curvature at any point (x, y) on the curve $y = \frac{1}{2}a\left(e^{\frac{s}{a}} + e^{-\frac{x}{a}}\right)$ (Creating)

Solution: Given $y = \frac{1}{2}a\left(e^{\frac{s}{a}} + e^{-\frac{x}{a}}\right) = a\cos h\left(\frac{x}{a}\right) = a\sin h\left(\frac{x}{a}\right).\frac{1}{a} = \sin h\left(\frac{x}{a}\right);\qquad \frac{d^2y}{dx^2} = a$

$$\cos h\left(\frac{x}{a}\right).\frac{1}{a} \therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx}} = \frac{\left[1+\sin h^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\cos\left(h\ \frac{x}{a}\right)}\ x\ a \quad = \frac{\left[\cos h^2\left(\frac{x}{a}\right)\right]^{\frac{3}{2}}}{\cos h\ \left(\frac{x}{a}\right)}.a \ = a\cos h^2\left(\frac{x}{a}\right)$$

13. Summarize the formula for radius of curvature is Cartesian form. (Understanding)

Solution: $\therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx}}$

14. Find the radius of curvature at (3, 10) on the curve $x\,y = 30$. (Remembering)

Solution: $x\,y = 30$ $\quad x\frac{dy}{dx} + \ y = 0\,\frac{dy}{dx} = \frac{-y}{x} \Rightarrow \left(\frac{dy}{dx}\right)_{(3,10)} \ = \ \frac{-10}{3}$

$$\frac{d^2y}{dx^2} = -\left[\frac{x\frac{dy}{dx}-y}{x^2}\right]\left(\frac{d^2y}{dx^2}\right)_{(3,10)} = -\left[\frac{3\left(\frac{-10}{3}\right)-10}{9}\right] = \frac{20}{9} \qquad \therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx}} = \frac{9}{20}\,\frac{109^{\frac{3}{2}}}{9^{\frac{3}{2}}}$$

15. Find 'ρ' for the curve $y = c\cos\text{h}\frac{x}{c}$ at any point (x, y). (Remembering)

Solution: $\frac{dy}{dx} = \frac{1}{c}\sin h\frac{x}{c}X\ c\ ;$ $\qquad \frac{d^2y}{dx^2} = \frac{1}{c}$

$$\cos h\ \frac{x}{a} \therefore \rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx}} = \frac{\left[1+\sin h^2\frac{x}{c}\right]^{\frac{3}{2}}}{\frac{1}{c}\cos h\ \frac{x}{c}} = \frac{c\ \left(\cos h^2\frac{x}{c}\right)^{\frac{3}{2}}}{\cos h\ \frac{x}{c}} = c.\frac{\cos h^3\frac{x}{c}}{\cos h\ \frac{x}{c}} = c\cos h\ \frac{x}{c}$$

16. Determine the radius of curvature at any point on $\ y = c\log\sec\left(\frac{x}{c}\right)$ (Evaluating)

Solution: $y' = c.\frac{1}{\sec\left(\frac{x}{c}\right)}.\sec\frac{x}{c}.\tan\frac{x}{c}.\frac{1}{c} = \ tan\ \frac{x}{c}y'' = \frac{1}{c}se^{2}\frac{x}{c}$

$$\rho\ at\ (\ 0,a) = \ a \qquad \therefore curvature = \ \frac{1}{\rho} = \frac{1}{a} \ = \frac{1}{a}.\frac{1}{\cos h^2} \therefore curvature\ at\ (0,a) is\ \frac{1}{a}$$

17. Prove that the radius of curvature for $y = e^x$ at the point where it cuts the Y-axis is $2\sqrt{2}$. (Evaluating)

To find the point, put $x = 0$, $\therefore$ y = 1 the point is (0,1). $y = e^x$; $\quad y' = e^x$; $\quad y'' = e^x$

$$\rho = \frac{(1+y^2)}{y''} = \ \frac{(1+e^{2x})^{3/2}}{ex}\,(\rho)_{(0,1)= 2\sqrt{2}}$$

18. What is the radius of curvature at (3, 4) on $x^2 + y^2 = 25$? (Remembering)

Solution: Since the given curve is a circle of radius 5 unit, and we know that the radius that the radius of curvature of a circle is equal to the radius of the given circle which 5, $\therefore \rho = 5$.

19. Find the radius of curvature at $x = 1$ on $x = \ \frac{\log x}{x^2}$ (Remembering)

Solution: $y' = \ \frac{1-log\ x}{x^2}\,y'' = \ \frac{2-log\ x-3}{x^3};\,(y')_{x\,=\,1} = 1;\ \left(y''\right)_{x\,=\,1} = -3;\ \rho = \frac{(1+1)^{3/2}}{-3} = \frac{2\sqrt{2}}{3}$

20. Show that the radius of curvature at any point on x^2 = 4 by is 2 b $\sec^2\theta$ where tan θ is the gradient of the curve at that point.(Understanding)

Solution: $\frac{dy}{dx} = \frac{x}{2b}$; $\quad \frac{d^2y}{dx^2} = \ \frac{1}{2b}$ $\qquad$ Given gradient t = tan θ

$a\frac{dy}{dx} = \tan\theta \qquad \frac{x}{2b} = \tan\theta \ \ , \ \ x = 2b\tan\theta;\ \rho = \frac{(1+y)^2}{y''} = \frac{(1+\ tan^2\theta)}{1/2\ b} = 2\ b\sec^3\theta$

21. What is average of curvature? Solution: $\quad \frac{d\psi}{ds} = \frac{1}{\rho}$ (Remembering)

22. Which is the equation of the circle of curvature. (Remembering)

Solution: $(x-\bar{x})^2 + (y-\bar{y})^2 = \rho^2$

23. The radius and centre of curvature of a curve at a point P are 3 and (0,5) respectively. What is the circle of curvature at the point P? (Remembering)

The equation of circle of curvature is $(x-\bar{x})^2+(y-\overline{y})^2=\rho^2$,Therefore at the given point P is $(x-0)^2+(y-5)^2=9$

24.Define evolutes and in volute of C. (Remembering)

Solution: The locus C of the centre of curvatures for a curve 'c' is called its evolutes. The curve 'c' is called an in volute of C.

25. State any two properties of an evolutes of a curve. (Remembering)

Solution:

(i)The normal at any point of a curve touches the evolutes' at the corresponding centre of curvature.

(ii)The length of an arc of the evolutes is equal to the difference between the radii of curvature at the points on the original curve corresponding to the extremities of the arc.

26. Define evolutes of a curve. (Remembering)

The locus of the centre of curvature of a curve is called evolutes of a curve.

PART B (QUESTIONS UNDER CO3)

1. What the radius of curvature at the point $\left(\frac{3a}{2},\frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. (Remembering)
2. Develop the radius of curvature at the point (x, y) on y = clog sec(x/c). (Applying)
3. Determine the radius of curvature at the point (a, 0) on the curve $xy^2 = a^3 - a^2x$. (Evaluating)
4. Find the radius of curvature at the point θ on the curve $x = 3a\cos\theta - a\cos3\theta$, $y = 3a\sin\theta - a\sin3\theta$ (Remembering)
5. Estimate the radius of curvature at the point 't' on $x = e^t\cos t$, $y = e^t\sin t$. (Creating)
6. Find the radius of curvature at the point $(a\cos^3\theta, a\sin^3\theta)$ on $x^{2/3} + y^{2/3} = a^{2/3}$. (Remembering)
7. Formulate the radius of curvature at the point θ on $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. (Creating)
8. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)4a\cos\theta/2$. (Evaluating)
9. If ρ is the radius of curvature at any point (x, y) on $y=\frac{ax}{a+x}$, Prove that $\left(\frac{2\rho}{a}\right)^{2/3}=\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2$ (Evaluating)
10. Find the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $\left(\frac{a}{4},\frac{a}{4}\right)$. (Remembering)
11. Solve the circle of curvature of the parabola $y^2 = 12x$ at (3,6). (Creating)
12. Find the centre and circle of curvature of the curve $xy = c^2$ at (c,c).
13. Find the evolute of the curves: (a) $y^2 = 4ax$, (b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, (c) $x^2 = 4ay$,
14. (d) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, (e) $x = a\cos\theta$, $y = b\sin\theta$, (f) $xy = c^2$, (g) $x^{2/3} + y^{2/3} = a^{2/3}$. (Remembering)

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UNIT IV - FUNCTIONS OF SEVERAL VARIABLES
PART A (QUESTIONS UNDER CO4)

1. Find du/dt if $u = x^3y^4$. Where $x = t^3$, $y= t^2$. (Remembering)

   $du/dt = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = 3x^2 3t^2 + 4x^3y^3\, 2t = 9\,t^{16} + 8t^{16} = 17t^{16}$.

2. Find dy/dx if $x^3 + y^3 = 3axy$.(Remembering)

   Given $f(x, y) = x^3 + y^3 - 3axy$. $\partial f/\partial x = 3x^2 - 3ay$. $\partial f/\partial y = 3y^2 - 3ax$.

   $dy/dx = -(\partial f/\partial x) / (\partial f/\partial y) = -(3x^2 - 3ay) / (3y^2 - 3ax) = (ay - x^2) / (y^2 - ax)$.

3. Find the Taylor's series expansions of x^y near the point (1, 1) up to the first degree terms. (Remembering)

   The Taylor's series expansion is

   $f(x,y) = f(a,b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \dots\dots$

   $f(x, y) = x^y$ $f(1, 1) = 1$

   $f_x(x,y) = yx^{y-1}$ $f_x(1,1) = 1.1^{1-1} = 1$

   $f_y(x,y) = x^y.\log x$ $f_y(1,1) = 1^1.\log 1 = 0$

   $\therefore\ f(x,y) = 1 + \frac{1}{1!}[(x-1)(1) + (y-1)(0)] + \cdots$ (i.e)$f(x, y) = 1 + x - 1 = x$.

4. Find the Taylor's series expansions of $e^x \sin y$ near the point $(-1,\pi/4)$up to the first degree terms. (Remembering)

   The Taylor's series expansion is

   $f(x, y) = f(a, b) + \frac{1}{1!}\left[(x-a)f_x(a,b) + (y-b)f_y(a,b)\right] + \dots\dots$

   $$fx(x,y) = e^x \sin y\ f\left(-1, {}^{\pi}/_{4}\right) = \frac{1}{\sqrt{2}.e}$$

   $f(x, y) = \frac{1}{\sqrt{2}.e} + \frac{1}{1!}\left[(x+1)\frac{1}{\sqrt{2}.e} + \left(y - {}^{\pi}/_{4}\right)\frac{1}{\sqrt{2}.e}\right] + \dots\dots$

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (Understanding)

   Given $u(x,y,z) = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}\, u(tx,ty,tz) = \frac{tx}{ty} + \frac{ty}{tz} + \frac{tz}{tx} = t^{\circ}.u(x,y,z)$

   $u(x,y,z)$ is homogeneous in x, y, z with degree n = 0, By Euler's theorem, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = n.u = 0$

6. If $u = \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (Apply)

   $u = \frac{y}{z} + \frac{z}{x}\frac{\partial u}{\partial x} = 0 - \frac{z}{x^2}$; $\frac{\partial u}{\partial y} = \frac{1}{z} + 0$; $\frac{\partial u}{\partial z} = \frac{-y}{z^2} + \frac{1}{x}$; $\frac{\partial u}{\partial x}x = -\frac{z}{x}$; $\frac{\partial u}{\partial y}y = \frac{y}{z}$;

   $$\frac{\partial u}{\partial z}z = \frac{-y}{z} + \frac{z}{x} \qquad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -\frac{z}{x} + \frac{y}{z} - \frac{y}{z} + \frac{z}{x} = 0$$

7. Define maximum . (Remembering)

   A function f(x,y) is said to have a relative maximum at (a, b) if f(a, b) > f(a+ h, b+ k) for all values of h and k.

8. Define minimum. (Remembering)

   A function f(x,y) is said to have a relative minimum at (a, b) if f(a, b) < f(a +h, b+ k) for all values of h and k.

9. Define saddle point. (Remembering)

   The point at which f(x, y) is neither a maximum nor a minimum is called saddle point.

10. Define functionally dependent. (Remembering)

    u, v are said to be functionally dependent functions of x, y if $\partial(u,v)/\partial(x,y) = 0$.

11. If $z = e^{ax+by}f(ax-by)$, show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.(Apply)

    Let u = ax+by, v = ax-by. Therefore, $z = e^u f(v)$.

    $\partial z/\partial x = \frac{\partial z}{\partial u}\frac{du}{dx} + \frac{\partial z}{\partial v}\frac{dv}{dx} = e^u f(v).a + e^u f'(v).a = az + a\,e^u f'(v)$.

    Therefore, $b(\partial z/\partial x) = abz + abe^u f'(v)$. Also $a(\partial z/\partial y) = abz - abe^u f'(v)$.

    Now $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

12. If $x^3 + 3x^3y + 6xy^2 + y^3 = 1$. Find dy/dx. (Understanding)

    Let $f(x,y) = {}^{x3} + 3^{x3}y + 6x^{y2} + {}^{y3} - 1$.

    Therefore, $dy/dx = -\frac{\partial f}{\partial x} / \frac{df}{dy}$. Now $\frac{\partial f}{\partial x} = 3x^2 + 6xy + 6y^2$; $\frac{\partial f}{\partial x} = 3x^2 + 12xy + 3y^2$

    Therefore, $dy/dx = -(3x^2 + 6xy + 6y^2)/(3x^2 + 12xy + 3y^2) = -(x^2 + 2xy + 2y^2)/(x^2 + 4xy + y^2)$

13. In Lagrangian multiplier method to find the values of x, y, z for which f(x, y, z) can have a conditional extremum, we have to construct the auxiliary function F(x, y, z) as …. (f(x, y, z) + λ g(x, y, z)). (Remembering)

14. If z = log(x^2+xy+y^2), show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2$.. .(Apply)

Given, z = log(x^2+xy+y^2).Then,

$$\frac{\partial z}{\partial x}=\frac{(2x+y)}{(x^2+xy+y^2)} \Rightarrow x\frac{\partial z}{\partial x}=\frac{(2x^2+xy)}{(x^2+xy+y^2)}------\longrightarrow \text{I}$$

$$\frac{\partial z}{\partial y}=\frac{(2y+x)}{(x^2+xy+y^2)} \Rightarrow y\frac{\partial z}{\partial y}=\frac{(2y^2+xy)}{(x^2+xy+y^2)}------------------\longrightarrow \text{II}$$

$$+\text{ II}\Longrightarrow x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{(2x^2+xy)}{(x^2+xy+y^2)}+\frac{(2y^2+xy)}{(x^2+xy+y^2)}=\frac{(2x^2+xy+2y^2+xy)}{(x^2+xy+y^2)}=\frac{2(x^2+xy+y^2)}{(x^2+xy+y^2)}=2.$$

Hence, $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=2.$

15. Find the stationary points of the function f(x, y) = x^3-y^3-3xy. (Evaluate)

Given, f(x, y) = x^3-y^3-3xy. Then,

$$\frac{\partial f}{\partial x}=0\Rightarrow 3x^2-3y=0 \Rightarrow x^2-y=0 \Rightarrow x^2=y--------\longrightarrow \text{I}$$

$$\frac{\partial f}{y}=0\Rightarrow -3y^2-3x=0 \Rightarrow y^2+x=0\,.$$

From I , it follows that $(x^2)^2+x=0\Rightarrow x^4+x=0 \Rightarrow x(x^3+1)=0\Rightarrow x=0\,\&x^3=-1\Rightarrow x=0\,\&\,x-$ -1. When x =0, I becomes y = 0 When x =-1, I becomes

y = 1.Therefore the stationary points are (0,0) & (-1, 1).

16. Write the necessary and sufficient condition for maxima and minima of a function of two variables. (Remembering)

Necessary conditions: The necessary conditions for f(x, y) to have a maximum or minimum at (a, b) are that $\frac{\partial f}{\partial x}=0$ & $\frac{\partial f}{\partial y}=0$ at (a, b). These conditions are not sufficient for f(x, y) to possess an extremum.

Sufficient conditions:

Let r = f_{xx}(a, b), s = f_{xy}(a, b) and t = f_{yy}(a, b). The function f(x, y) will possess an extremes at (a, b) if f_x(a, b) = 0, f_y(a, b) = 0 & rt-s^2> 0, r>0 & r<0. Hence, if rt-s^2> 0, then f(x, y) has a maximum or minimum at (a, b) according as r<0 or r>0. If rt-s^2< 0, then there is no maximum and minimum at (a, b).

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PART B (QUESTIONS UNDER CO4)

1. If u = (x - y)f(y/x), find $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. (Evaluate)

2. If u = tan $^{-1}\left[\frac{x^2+y^2}{x+y}\right]$ Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$. (Apply)

3. If g(x ,y) = $\psi(u,v)$ where u = x^2–y^2& v = 2xy, prove that $\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}=4(x^2+y^2)\left(\frac{\partial^2\psi}{\partial u^2}+\frac{\partial^2\psi}{\partial v^2}\right)$. (Apply)

4. Given the transformations u = e^xcosy & v = e^xsiny and that ϕ is a function of u and v and also of x and y, prove that $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=(u^2+v^2)\left(\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial v^2}\right)$. (Apply)

5. If u = f $\left(\frac{y-x}{xy},\frac{z-x}{xz}\right)$, show that $x^2\frac{\partial u}{\partial x}+y^2\frac{\partial u}{\partial y}+z^2\frac{\partial u}{\partial z}=0$

5. If u = x log xy, where x^3 + y^3 + 3xy = 1, find du/dx. (Understanding)

6. Obtain terms up to the third degree in the Taylor series expansion of e^x sin y around the point (1, π/2). (Evaluate)

7. Expand the function sin xy in powers of x – 1 and y – π/2 up to second degree terms. (Understand)

8. Obtain the Taylor series of $x^3 + y^3 + xy^2$ at (1, 2). (Understand)

9. If x + y + z = u, y + z = uv, z = uvw, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$. (Evaluate)

Find the extreme values of the following functions: (a) $x^3 + y^3 - 12x - 3y + 20$, (b) $x^3 + y^3 - 3axy$, (c) $x^3y^2(12 - x - y)$. (Evaluate)

10. Find the minimum values of x^2yz^3 subject to the condition $2x + y + 3z = a$. (Evaluate)

11. A rectangular box open at the top, is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction. (Evaluate)

12. Find the dimensions of a rectangular box, open at the top with maximum volume and the surface area is 27cm^2. The temperature u(x, y, z) at any point in space is $u = 400xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$. (Evaluate)

13. Find the maximum volume of the largest rectangular parallel piped than can be inscribed in a ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (Evaluate)

14. Find the maximum value of $x^m y^n z^p$ when x + y + z = a (Evaluate)

15. Find the minimum value of $x^2 + y^2 + z^2$ with the constraint x y + y z + z x= 3a^2. (Evaluate)

16.If $u = \log(x^3+y^3+z^3-3xy)$ prove that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{9}{(x+y+z)^2}$. (Apply)

UNIT V - VECTOR CALCULUS
PART A (QUESTIONS UNDER CO5)

1. Find the gradient of $x^2+y^3+z^4$ at the point (1,-1, 1). (Evaluate)

Solution: Given φ = $x^2+y^3+z^4$ at (1,-1,1), $grad\phi = \nabla\phi = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{dz}$

ie, I $(2x)+j(3y^2)+k(4z^3)$ at (1,-1,1), hence ∇φ = i(2)+j(3)+k(4) = 2i+3j+4k

2. Find the magnitude and direction in which the following function $2yz+3x-z^2$ at (2,1,3) (Evaluate)

Solution: $grad\phi = \nabla\phi = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{dz}$ = 3i+2zj +(2y-2z)k at (2,1,3)

∇φ = 3i+ (2x3)j+(2-6)k = 3i+6J-4k , |∇φ| = √61

3. Find the directional derivative of φ = x y+ y z+ z x at (1,2,0) in the direction of i+2j+2k. (Evaluate)

Solution: Given φ = x y+ y z+ z x at (1,2 ,0) in the direction of $\vec{a} = i + 2j + 2k$

Directional derivative $= \frac{\nabla\phi.\vec{a}}{|\vec{a}|}$, $\nabla\phi = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{dz}$

⇒ i(y+0+z)+j(x+z+0)+k(0+y+x) at (1,2,0), $\nabla\phi$ =2i+j+3k ,

$\nabla\phi.\vec{a} =$ (2i+j+3k). (i+2j+2k) = 10, $|\vec{a}|$ = 3, hence Directional derivative = 10/3

4. Find the angle between the following surfaces $x^2+y^2+z^2 = 9$ and $z= x^2+y^2-3$ at the point (2,-1,2). (Evaluate)

Solution:φ = $x^2+y^2+z^2$ = 9 , ∇φ = i(2x)+j(2y)+k(2z), ∇φ at (2,-1,2) = $\vec{a} = 4i - 2j + 4k$

φ = z- x^2+y^2+3, ∇φ = -4i+2j+k, $\vec{b}$ = -4i+2j+k, now $\vec{a} . \vec{b}$ = -16,

$|\vec{a}| = \sqrt{6}$ $|\vec{b}| = \sqrt{21}$, now angle between the lines is $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}$

Hence cos θ = -8/3x√21 ⇒θ = cos $^{-1}$ (-8/3√21)

5. Find the divergence of $\vec{F} = x^2 i + y^3 j + z^4 k$. (Evaluate)

Solution: $f_1 = x^2$, $f_2 = y^3$, $f_3 = z^4$, now $div\vec{F} = \nabla.\vec{F} = \dfrac{\partial f_1}{\partial x} + \dfrac{\partial f_2}{\partial y} + \dfrac{\partial f_3}{\partial z} = 2x + 3y^2 + 4z^3$

6. If $\vec{F} = x^2\vec{i} + y^2\vec{j}$ evaluate $\int_c \vec{F}.d\vec{r}$ along the line y = x from (0,0) to (1,1).

Solution: $\int_c \vec{F}.d\vec{r} = \int_c (x^2\vec{i} + y^2\vec{j}).(dx\vec{i} + dy\vec{j} + dz\vec{k}) = \int_c x^2 dx + y^2 dx$, put y = x and

dy = dx $\Rightarrow \int_c \vec{F}.d\vec{r} = 2\int_0^1 x^2 dx$ = 2/3

7. What is the unit normal to the surface φ = c? (Remembering)

Solution: Given φ = c, ie, φ(x, y, z) = c, hence e unit normal = ∇φ/|∇φ|

8. Define grade φ and div $\vec{F}$. (Remembering)

Solution: Let φ(x, y, z) be a scalar point function and is continuously differentiable then the vector $\nabla\phi = \left(\vec{i}\dfrac{\partial}{\partial x} + \vec{j}\dfrac{\partial}{\partial y} + \vec{k}\dfrac{\partial}{\partial z}\right)\phi = \left(\vec{i}\dfrac{\partial\phi}{\partial x} + \vec{j}\dfrac{\partial\phi}{\partial y} + \vec{k}\dfrac{\partial\phi}{\partial z}\right)$ is called the gradient of the scalar function φ and is written as grade φ.

9. Define Div $\vec{F}$. (Remembering)

Solution: If $\vec{F}$ is a vector function, the scalar product of the vector operator ∇ and $\vec{F}$ gives a scalar which is called the divergence of $\vec{F}$ or $\nabla.\vec{F}$

10. For what value of k is the vector $r^k\vec{r}$ solenoid? (Understanding)

Solution: Let $\vec{F} = r^k\vec{r} = r^k(x\vec{i} + y\vec{j} + z\vec{k}), \nabla.\vec{F} = \dfrac{\partial}{\partial x}(r^k x) = \sum\left[r^k + xkr^{k-1}\dfrac{\partial r}{\partial x}\right]$

$$\Rightarrow \sum\left[r^k + xkr^{k-1}\frac{x}{r}\right] = \sum\left[r^k + kr^{k-2}x^2\right] = 3r^k + kr^{k-2}(r^2) = (3+k)r^k$$

If k = -3 we get $\nabla.\vec{F}$ = 0, hence $r^k\vec{r}$ solenoid only if k = -3.

11. Prove that $\nabla(r^n) = nr^{n-2}\vec{r}$. (Remembering)

Solution:

$$\nabla(r^n) = \sum\vec{i}\,\partial/\partial x(r^n) = \sum\vec{i}nr^{n-1}\partial r/\partial x = \sum\vec{i}nr^{n-1}(x/r) = nr^{n-2}(x\vec{i} + y\vec{j} + z\vec{k}) = nr^{n-2}\vec{r}$$

12. Find a unit normal vector to the surface $x^2 + y^2 - z = 10$ at (1, 1, 1). (Evaluate)

Solution: Given $\phi = x^2 + y^2 - z = 10$

$$\nabla\phi = \vec{i}\partial\phi/\partial x + \vec{j}\partial\phi/\partial y + \vec{k}\partial\phi/\partial z = 2x\vec{i} + 2y\vec{j} - \vec{k}.$$

$$\nabla\phi(1,1,1) = 2\vec{i} + 2\vec{j} - \vec{k}\ ,\ |\nabla\phi| = \sqrt{4+4+1} = \sqrt{9} = 3 \Rightarrow \vec{n} = \frac{\nabla\phi}{|\nabla\phi|} = 2\vec{i} + 2\vec{j} - \vec{k}/3$$

13. If $\vec{F}=x^3\vec{i}+y^3\vec{j}+z^3\vec{k}$, find div curl $\vec{F}$. (Evaluate)

Solution: Curl $\vec{F}$ = $\nabla x\vec{F}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^3 & y^3 & z^3 \end{vmatrix}=\vec{i}(0-0)-\vec{j}(0-0)+\vec{k}(0-0)=0$,

$\therefore$ Div curl $\vec{F}=\nabla.\nabla\times\vec{F}=0$.

14. If $\vec{v}=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+\lambda z)\vec{k}$ is solenoid find the value of λ. (Evaluate)

Solution: Given $\nabla.\vec{v}=0\Rightarrow\dfrac{\partial}{\partial x}(x+3y)+\dfrac{\partial}{\partial y}(y-2z)+\dfrac{\partial}{\partial z}(x+\lambda z)=0$,

$1+1+\lambda=0\Rightarrow\lambda=-2$

15. What is the condition for the vector point function $\vec{F}$ to be solenoidal? (Remembering)

Solution: The condition for the vector point function $\vec{F}$ to be solenoid is div $\vec{F}$ =0.

16. Prove that $yz\vec{i}+zx\vec{j}+xy\vec{k}$ is irrigational. (Remembering)

Solution: Let $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$

$$\nabla x\vec{F}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}=\sum\vec{i}(x-x)=\sum 0\vec{i}=0\vec{i}+0\vec{j}+0\vec{k}\Rightarrow\nabla\times\vec{F}=0,$$

Hence the given vector is irrotational.

17. State Gauss divergence theorem. (Remembering)

Solution: The surface integral of the normal component of a vector function taken over a closed surface s enclosing a volume v is equal to the volume integral of the divergence of $\vec{F}$ taken through the volume v.

i.e., $\iint_s \vec{F}.n\,ds=\iiint_v \nabla.\vec{F}\,dv$.

18. State Green's theorem. (Remembering)

Solution: If u and v are continuous functions of x and y having partial derivatives in a region R enclosed by a simple closed curve C, then $\int_c (udx+vdy)=\iint_R \left(\dfrac{\partial v}{\partial x}-\dfrac{\partial u}{\partial y}\right)dxdy$.

19. State Stoke's theorem. (Remembering)

Solution: If $\vec{F}$ is a continuous vector function with continuous partial derivatives on an open surface S bounded by a simple closed curve C, then $\int_c \vec{F}.d\vec{r}=\iint\left(\nabla\times\vec{F}\right).n\,ds$.

20. If $\nabla^2\phi=0$, prove that $\nabla\phi$ is both solenoid and irrigational. (Remembering)

Solution: Given $\nabla^2\phi=0\Rightarrow\nabla.\nabla\phi=0\Rightarrow\nabla\phi$ is solenoid. ($\Theta\,\nabla.\vec{F}=0\Rightarrow\vec{F}$ is solenoid)

Now $\nabla^2\phi=0\Rightarrow\nabla.\nabla\phi=0\Rightarrow\nabla\phi=0$, $\therefore$ $\nabla\phi=0\Rightarrow\nabla\times\nabla\phi=0\Rightarrow\nabla\phi$ is irrigational.

21. If $\vec{A}$ and $\vec{B}$ are irrigational vectors, show that $\vec{A}\times\vec{B}$ is solenoid. (Remembering)

Solution: $\vec{A}$ and $\vec{B}$ are irrigational vectors.

$\therefore\ \nabla\times\vec{A}=0$ and $\nabla\times\vec{B}=0$ ……..(1). To show that $\vec{A}\times\vec{B}$ is solenoid. i.e., $\nabla.(\vec{A}\times\vec{B})=0$

WKT $\nabla.(\vec{A}\times\vec{B})=(\nabla\times\vec{A}).\vec{B}-(\nabla\times\vec{B}).\vec{A}$ = 0-0 =0 by (1) $\Rightarrow\vec{A}\times\vec{B}$ is solenoid.

PART – B (QUESTIONS UNDER CO5)

1. Given $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Then prove that $r^n\vec{r}$ is solenoid only when n = -3, but irrotational for all values of n. (Evaluate)
2. Find a and b such that the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at (1,-1, 2). (Evaluate)
3. Verify Gauss divergence theorem for $\vec{F}=x^2\hat{i}+y^2\hat{j}+z^2\hat{k}$ where S is the surface of the cuboids formed by the planes x = 0,x = a,y = 0,y = b, z = 0,z = c . (Apply)
4. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$,then prove that $div(grad(r^n))=n(n+1)r^{n-2}$.Hence deduce that $div[grad\left(\frac{1}{r}\right)]=0$. (Evaluate)
5. Prove that $\nabla.(\vec{A}\times\vec{B})=\vec{B}.curl\vec{A}-\vec{A}.curl\vec{B}$.Hence deduce that $\vec{A}\times\vec{B}$ is solenoid where $\vec{A}$ and $\vec{B}$ is irrigational. . (Evaluate)
6. Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative field. Find the scalar potential. (Evaluate)
7. Evaluate $\int_c\vec{f}.\vec{dr}$ where $\vec{f}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ along the straight line joining (1,-2,1) and (3,2,4). . (Evaluate)
8. Evaluate $\iint_s\vec{f}.\hat{n}dS$ where $\vec{f}=(x+y^2)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the surface of the plane 2x + y + 2z = 6 in the first octant. (Evaluate)
9. If $\vec{A}=2xy\hat{i}+(x^2+2yz)\hat{j}+(y^2+1)\hat{k},$ find the value of $\int_c\vec{A}.\vec{dr}$ around the unit circle with centre at the origin in the x y plane. (Remembering)
10. a and b such that the Find surfaces $ax^3-by^2z=(a+3)x^2$ and $4x^2y-z^3=11$ cut orthogonally at (2,-1,-3).
11. Prove that $curl(curl\vec{F})=grad(div\vec{F})-\nabla^2\vec{F}$. (Evaluate)
12. Verify Gauss divergence theorem for $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ taken over the cube formed by the planes x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. (Apply)
13. Verify Gauss divergence theorem for $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$, taken over the rectangular parallelepiped $0\le x\le a$, $0\le y\le b$, $0\le z\le c$. . (Apply)
14. Prove that $\vec{F}=(y^2\cos x+z^3)\hat{i}+(2y\sin x-4)\hat{j}+3xz^2\hat{k}$ is irrigational and find the scalar potential ϕ (Evaluate)

15. Prove that $\vec{F} = (y^2 + 2xz^2)\hat{i} + (2xy - z)\hat{j} + (2x^2z - y + 2z)\hat{k}$ is irrigational and find the scalar potential ϕ.
(Evaluate)

16. Find $\int_c \vec{F}.\vec{dr}$ where $\vec{F} = (2y+3)\hat{i} + xz\hat{j} + (yz - x)\hat{k}$ along the line joining the points (0, 0, 0) to (2,1,1). . (Remembering)

17. Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ in the rectangular region in the x y plane bounded by the lines x = 0,x = a,y = 0 and y = b. (Apply)

18. Apply Green's theorem in the plane to evaluate $\int_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by x = 0,y = 0 and x + y = 1. $\nabla\phi = 2xyz^3\,\vec{i} + x^2z^3\,\vec{j} + 3x^2yz^2\,\vec{k}$,find $\phi(x, y, z)$, if $\phi(1,-2,2) = 4$. (Evaluate)

19. Find the directional derivative of $\phi = x^2 + yz + 4z^2$ at P(1,-2,-1) in the direction of PQ where Q is (3,-3,-2).
(Remembering)

20. If $\vec{F} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$,evaluate $\int_c \vec{F}.\vec{dr}$ from (0,0,0) to (1,1,1) along the curve x = t,y = t^2,z t^3.
(Apply)

21. If S is any closed surface enclosing a volume V and if $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$, prove that $\int_s \vec{A}.\hat{n}\,ds$ = (a+b+c) v.
(Apply)

22. Verify Green's theorem for $\int_c (xy + y^2)dx + x^2dy$ where C is the boundary of the common area between y = x^2 and y =x. (Apply)

23. Evaluate by Stoke's theorem $\int_c \vec{F}.\vec{dr}$ where $\vec{F} = \sin z\,\hat{i} - \cos x\,\hat{j} + \sin y\,\hat{k}$ where C is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1, z = 3.$ (Evaluate)

24. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ in the rectangular region in the x y plane bounded by the lines x = $\pm$ a, y = 0 and y = b. (Apply)

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