K.S.R. COLLEGE OF ENGINEERING (Autonomous) –TIRUCHENGODE.

Vision of the Institution

- We envision to achieve status as an excellent educational institution in the global knowledge hub, making self-learners, experts, ethical and responsible engineers, technologists, scientists, managers, administrators and entrepreneurs who will significantly contribute to research and environment friendly sustainable growth of the nation and the world.

Mission of the Institution

- To inculcate in the students self-learning abilities that enable them to become competitive and considerate engineers, technologists, scientists, managers, administrators and entrepreneurs by diligently imparting the best of education, nurturing environmental and social needs.
- To foster and maintain a mutually beneficial partnership with global industries and Institutions through knowledge sharing, collaborative research and innovation.

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Vision of the Department

- To create ever green professionals for software industry, academicians for knowledge cultivation and researchers for contemporary society modernization.

Mission of the Department

- To produce proficient design, code and system engineers for software development.
- To keep updated contemporary technology and fore coming challenges for welfare of the society.

Programme Educational Objectives (PEOs)

PEO1 : Figure out, formulate, analyze typical problems and develop effective solutions by imparting the idea and principles of science, mathematics, engineering fundamentals and computing.

PEO2 : Competent professionally and successful in their chosen career through life-long learning.

PEO3 : Excel individually or as member of a team in carrying out projects and exhibit social needs and follow professional ethics.

DATE **COURSE FACULTY** **H.O.D** **PRINCIPAL**

K.S.R. COLLEGE OF ENGINEERING (AUTONOMOUS) - TIRUCHENGODE.

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Subject Name: DISCRETE MATHEMATICS

Subject Code: 18MA243 **Year/Semester: I / II**

Course Outcomes: On completion of this course, the student will be able to

CO-1	**Using mathematical techniques to solve logical problems.**
CO-2	**Construct algorithms and derive complexities.**
CO-3	**Acquire the knowledge of sets that are required for developing computational models.**
CO-4	**Perform computational operations associated with functions.**
CO-5	**Apply the concepts of Graph theory and Combinatorics in network algorithms**

Program Outcomes (POs) and Program Specific Outcomes (PSOs)

A. Program Outcomes (POs)

Engineering Graduates will be able to :

PO1	**Engineering knowledge:** Ability to exhibit the knowledge of mathematics, science, engineering fundamentals and programming skills to solve problems in computer science.
PO2	**Problem analysis:** Talent to identify, formulate, analyze and solve complex engineering problems with the knowledge of computer science. .
PO3	**Design/development of solutions:** Capability to design, implement, and evaluate a computer based system, process, component or program to meet desired needs.
PO4	**Conduct investigations of complex problems:** Potential to conduct investigation of complex problems by methods that include appropriate experiments, analysis and synthesis of information in order to reach valid conclusions.
PO5	**Modern tool Usage:** Ability to create, select, and apply appropriate techniques, resources and modern engineering tools to solve complex engineering problems.
PO6	**The engineer and society:** Skill to acquire the broad education necessary to understand the impact of engineering solutions on a global economic, environmental, social, political, ethical, health and safety.
PO7	**Environmental and sustainability:** Ability to understand the impact of the professional engineering solutions in societal and Environmental contexts and demonstrate the knowledge of, and need for sustainable development.
PO8	**Ethics:** Apply ethical principles and commit to professional ethics and responsibility and norms of the engineering practices.
PO9	**Individual and team work:** Ability to function individually as well as on multi-disciplinary teams.
PO10	**Communication:** Ability to communicate effectively in both verbal and written mode to excel in the career.
PO11	**Project management and finance:** Ability to integrate the knowledge of engineering and management principles to work as a member and leader in a team on diverse projects.
PO12	**Life-long learning:** Ability to recognize the need of technological change by independent and life-long learning.

B. Program Specific Outcomes (PSOs)

PSO1	Develop and Implement computer solutions that accomplish goals to the industry, government or research by exploring new technologies.
PSO2	Grow intellectually and professionally in the chosen field.

DATE **COURSE FACULTY** **H.O.D** **PRINCIPAL**

K.S.R. COLLEGE OF ENINGEERING, TIRUCHENGODE – 637 215

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

18MA243 - DISCRETE MATHEMATICS

FIRST YEAR - SECOND SEMESTER

<u>QUESTION BANK</u>

UNIT – I: PROPOSITIONAL CALCULUS

<u>PART-A</u>

1. State the truth of "If tigers have wings then the earth travels round the sun". (Remembering)
   <u>Answer</u>: Let P: Tigers have wings. "F" Q: The earth travels round the sun. "F"
   $\therefore$ The given statement is $P \to Q$, has the truth value "T"

2. Give the converse and the contra-positive of the implication "If it is raining, then I get wet"
   <u>Answer</u>: P: It is raining. Q: I get wet.
   $Q \to P$: (converse) If I get wet, then it is raining.
   $\neg Q \to \neg P$: (contra-positive) If I do not get wet, then it is not raining. (Understanding)

3. Define contra-positive of a statement. (Remembering)
   <u>Answer</u>: For any statement formula $P \to Q$, the statement formula $Q \to P$ is called its converse, $\neg P \to \neg Q$ is called its inverse, and $\neg Q \to \neg P$ is called its contra-positive.

4. Define functionally complete set of connectives and give an example. (Remembering)
   <u>Answer</u>: Any set of connectives in which every formula can be expressed as another equivalent formula containing connectives from this set is called functionally complete set of connective. Example:The set of connectives$\{\wedge, \neg\}$ and $\{\vee, \neg\}$ are functionally complete. $\{\neg\}$, $\{\wedge\}$,$\{\vee\}$ or $\{\wedge, \vee\}$ are not functionally complete.

5. Write the negation of the following proposition. "To enter into the country you need a passport or a voter registration card". (Evaluating)
   <u>Answer</u>: To enter into the country you need 'not' a passport or a voter registration card.

6. Define DNF. (Remembering)
   <u>Answer</u>: A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a disjunctive normal form (DNF) of the given formula.
7. Define CNF. (Remembering)
   <u>Answer</u>: A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive normal form of the given formula.
8. Define Min-terms (or) Boolean conjunctions. (Remembering)
   <u>Answer</u>: Let P and Q be two statement variables. The formulas $P\wedge Q$, $P\wedge\neg Q$, $\neg P\wedge Q$, $\neg P\wedge\neg Q$ are called min-terms or Boolean conjunctions of P and Q.
9. Define Max terms. (or) Boolean disjunction. (Remembering)
   <u>Answer</u>: For two statement variables P, Q the formulas $P \vee Q$, $P \vee \neg Q$, $\neg P \vee Q$,
   $\neg P \vee \neg Q$ are max terms or P and Q.

10. Define Inference theory. (Remembering)

<u>Answer</u>: The main function of logic is to provide rules of inference, or principles of reasoning. The theory associated with such rules is known as inference theory because it is concerned with the inferring of a conclusion from certain premises.

11. State the rules of inference theory. (Remembering)

<u>Answer</u>: Rule P: A premise may be introduced at any point in the derivation.

Rule T: A formula S may be introduced in a derivation if S is a tautologically implied by any one or more of the preceding formulas in the derivation.

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \to S$ from the set of premises alone.

12 .Define the term logically equivalent. (Remembering)

<u>Answer</u>: The propositions P and Q are called logically equivalent if $P \leftrightarrow Q$ is a tautology.

13. What is tautology? Give an example. (Remembering)

<u>Answer</u>: A statement that is true for all possible values of its propositional variables is called a tautology or universely valid formula or a logical truth. Q. Example: $P \vee \neg P$ is a tautology.

14. Show that $\uparrow$ is a minimal functionally complete set. (Remembering)

<u>Answer</u>: To prove $\uparrow$ is a functionally complete set.

(i) $\neg P \Leftrightarrow \neg P \vee \neg P \Leftrightarrow \neg(P \wedge P) \Leftrightarrow P \uparrow P \to (1)$

(ii) $P \wedge Q \Leftrightarrow \neg(P \uparrow Q) \Leftrightarrow (P \uparrow Q) \uparrow (P \uparrow P) \to (2)$

(iii) $P \vee Q \Leftrightarrow \neg(\neg P \wedge \neg Q) \Leftrightarrow \neg P \uparrow \neg Q$

$\Leftrightarrow (P \uparrow P) \uparrow (Q \uparrow Q) \to (3)$

By (1), (2) & (3) $\uparrow$ is a functionally complete set.

15. How can this English sentence be translated into a logical expression? (Remembering)

<u>Answer</u>: "You can access the Internet from campus only if you are a computer science major or you are not a freshman".

a: You can access the Internet from campus.

b: You are a computer science major.

c: You are a freshman.

$\neg c$: You are not a freshman. $\therefore a \to (b \vee \neg c)$.

16.Show that $\{\vee, \wedge\}$ is not functionally complete. (Remembering)

<u>Answer</u>: $\neg P$ cannot be expressed using the connectives $\{\vee, \wedge\}$. Since no such contribution of statement exist with $\{\vee, \wedge\}$ as input is T and the output is F.

17.Define valid argument or valid conclusion. (Remembering)

<u>Answer</u>: If a conclusion is derived from a set of premises by using the accepted rules of reasoning, then such a process of derivation is called a deduction or a formal proof and the argument or conclusion is called a valid argument or valid conclusion.

PART-B

1. Prove that $((P \to Q) \wedge (Q \to R)) \Rightarrow (P \to R)$ (Evaluating)
2. Find the principal conjunctive and principal disjunctive normal forms of the formula $S \Leftrightarrow (P \to (Q \wedge R)) \wedge (\neg P \to (\neg Q \wedge \neg R))$ (Remembering)
3. Using conditional proof prove that $\neg P \vee Q, \neg Q \vee R, R \to S \Rightarrow P \to S$ (Evaluating)
4. By using truth tables verify whether the following specifications are consistent: Whenever the system software is being upgraded users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files then the system software is not being upgraded. (Analyzing)
5. Without using truth tables, show that $Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a tautology. (Remembering)
6. Obtain the PDNF for $(P \wedge Q) \vee (\neg P \wedge Q) \vee (Q \wedge R)$ (Remembering)
7. Show that S is valid inference from the premises $P \to \neg Q, Q \vee R, \neg S \to P and \neg R$ (Remembering)
8. Without using truth tables, show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ (Remembering)
9. Obtain the pcnf of the formula S given by $(\neg P \to R) \wedge (Q \to P)$ and hence deduce the PDNF of S. (Remembering)
10. Show that $R \wedge (P \vee Q)$ is a valid argument from the premises $P \vee Q, Q \to R, P \to M, \neg M$. (Remembering)
11. Without using truth tables prove the equivalences. (Evaluating) $P \to (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \to R \text{ and } P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$.
12. Show that the premises $R \to \neg Q, R \vee S, S \to \neg Q, P \to Q, P$ are inconsistent. (Remembering)
13. Prove that $(\neg Q \to \neg P) \wedge (\neg R \to \neg Q) \Rightarrow (P \to R)$ (Evaluating)
14. Find the principal disjunctive and conjunctive normal forms of the formula $S \Leftrightarrow ((\neg Q \vee \neg R) \to \neg P) \wedge (Q \vee R) \to P$ (Remembering)
15. Without constructing the truth tables show that $A \vee C$ is not a valid consequence of the premises $A \leftrightarrow (B \to C), B \leftrightarrow (\neg A \vee \neg C), C \leftrightarrow (A \vee \neg B), B$ (Remembering)
16. Using derivation process prove that $S \to \neg Q, S \vee R, \neg R, (\neg R \leftrightarrow Q) \Rightarrow \neg P$ (Evaluating)
17. Without using truth table, find the principal conjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$ (Remembering)
18. Show that $R \vee S$ follows logically from the premises (Remembering) $(C \vee D), ((C \vee D) \to \neg H), \neg H \to (A \wedge \neg B) and (A \wedge \neg B) \to (R \vee S)$
19. Determine whether the compound proposition $\neg(q \to r) \wedge r \wedge (p \to q)$ is tautology or contradiction. (Analyzing)
20. Show that d can be derived from the premises $(a \to b) \wedge (a \to c), \neg(b \wedge c), (d \vee a)$ (Remembering)
21. Show that the expression $((P \vee Q) \wedge (P \to R) \wedge (Q \to R)) \to R$ is a tautology by the use of truth tables. (Remembering) .

UNIT – II: PREDICATE CALCULUS

PART-A

1. Let p(x) denotes the statement "$x > 4$". What are the truth values of P(5) and P(2)? (Remembering)

   Answer: We obtain the statement P(5), by setting $x = 5$ in the statement "$x > 4$". Hence P(5), which is the statement " $5 > 4$" is true. However, P(2), which is the statement "$2 > 4$ " is false.

2. Let Q(x, y) denote the statement "$x = y + 2$", what are the truth values of the prepositions Q(1, 2) and Q(2, 0). (Remembering)

   Answer: To obtain Q(1, 2). Set $x = 1$ and $y = 2$ in the statement Q(x, y). Hence, Q(1, 2) is the statement "$1 = 2 + 2$", which is false. The statement Q(2, 0) is the proposition "$2 = 0 + 2$", which is true.

3. Define Quantifiers. (Remembering)

   Answer: Certain declarative sentences involve words that indicate quantity such as "all, some, none or one". These words help determine the answer to the question "How many?". Since such words indicate quantity they are called quantifiers.

4. Define a compound statement function. (Remembering)

   Answer: A compound statement function is obtained by combining one or more simple statement functions by logical connectives.

5. Symbolize: For every x, there exists a y such that $x^2 + y^2 \geq 100$.

   Answer: $(\forall x)(\exists y)(x^2 + y^2 \geq 100)$ (Understanding)

6. Define the term Existential quantifier. (Remembering)

   Answer: The quantifier "some" is the Existential quantifier and we shall denote it by $\exists x$, which is a reversed E followed by x. It represents each of the following phrases:

   There exists an x such that ….

   There is an x such that ….

   For some x ….

   There is at least one x such that ….

   Some x is such that ….

7. Define "a simple statement function." With an example. (Remembering)

   Answer: A simple statement function:

   A simple statement function of on variable is defined to be an expression consisting of a predicate symbol and an individual variable. Such a statement function becomes a statement when the variable is replaced by the name of any object.

   Example: If "X is a teacher" is denoted by T(x), it is a statement function. If X is replaced by John, then "John is a teacher" is a statement.

8. What are the truth values of the propositions R(1, 2, 3) and R(0, 0, 1)? (Remembering)

   Answer: The proposition R(1, 2, 3) is obtained by setting $x = 1$, $y = 2$ and $z = 3$ in the statement R(x, y, z). We see that R(1, 2, 3) is the statement "$1 + 2 = 3$", which is true. Also note that R(0, 0, 1), which is the statement "$0 + 0 = 1$" is false.

9. Give the symbolic form of the statement "every book with a blue cover is a mathematics book".

   Answer: $\forall x\, (S(x)) \rightarrow P(x)$ where S(x) = x is every book with a blue cover &

   P(x) = Mathematics book.. (Understanding)

10. Rewrite the following using quantifiers. "Some men are genius". (Creating)

Answer: M(x) : x is a man G(x) : x is genius $(\exists x)(M(x) \wedge G(x))$

11. Write the following statement in the symbolic form: everyone who likes fun will enjoy each of these plays. (Understanding)

Answer: We write L(x) : x likes fun. P(x) : x is a play.
E(x, y) : x will enjoy y. Then $(\forall x)[L(x) \rightarrow (\forall y)(P(y) \rightarrow E(x, y)]$.

12. Let P(x) : x is a person, F(x, y) : x is the father of y, M(x, y) : x is the mother of y write the predicate "x is the father of the mother of y". (Understanding)

Answer: $(\exists z)P(z) \wedge F(x, z) \wedge M(z, y)$

13. Symbolize the expression "All the world loves a lover". (Understanding)

Answer: Let P(x) : x is a person L(x) : x is a lover R(x, y) : x loves y The required expression if $(x)(P(x) \rightarrow (y)(P(y) \wedge (L(y) \rightarrow R(x, y))$.

14. State the rules of inference theory. (Remembering)

Answer: Rule P: A given premise may be introduced at any stage in the derivation.
Rule T: A formula S may be introduced in a derivation if S is tautologically implied by any one or more of the preceding formulas in the derivation.
Rule CP: If we can derive S from R and a set of given premises, then we can derive $R \rightarrow S$ from the set of premises alone.

15. Write the following sentence in a symbolic form "Every one who is healthy can do all kinds of work". (Understanding)

Answer: H(x) : x is a healthy person, H(y) : y is a kind of work:D(x, y) : x can do y
$\Rightarrow \quad (x)(y)[H(x) \wedge H(y) \rightarrow D(x, y)]$

16. Symbolize the statement. "Given any positive integer, there is a greater positive integer".

Answer: P(x) : x is a positive integer, G[x, y] : x us greater than y.
$\Rightarrow x(P(x) \rightarrow (\exists y)(P(y) \wedge G(y, x))$. (Understanding)

PART-B

1. Use indirect method of proof to show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (Remembering)
2. Prove that $(\exists x)P(x) \rightarrow (x)Q(x) \Rightarrow (x)(P(x) \rightarrow Q(x))$ (Evaluating)
3. Use conditional proof to prove that $(x)(P(x) \rightarrow Q(x)) \Rightarrow (x)P(x) \rightarrow (x)Q(x)$. (Evaluating)
4. Prove that $(\exists x)(A(x) \vee B(x)) \Leftrightarrow (\exists x)A(x) \vee (\exists x)B(x)$. (Evaluating)
5. Show that $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$. (Remembering)
6. Is the following conclusion validly derivable from the premises given?
   If $(\forall x)(P(x) \rightarrow Q(x)), (\exists y)P(y), then\ (\exists z)Q(z)$ (Evaluating)
7. Verify the validity of the inference. If one person is more successful than another, then he has worked harder to deserve success. John has not harder than Peter. Therefore, John is not successful than Peter. (Evaluating)
8. Prove that $(\exists x)(P(x) \wedge S(x)), (\forall x)(P(x) \rightarrow R(x)) \Rightarrow (\exists x)(R(x) \wedge S(x))$ (Evaluating)
9. By indirect method, prove that $(\forall x)(P(x) \rightarrow Q(x)),\ (\exists x)P(x) \Rightarrow (\exists x)(Qx)$ (Evaluating)
10. Find the scope of the quantifiers and the nature of occurrence of the variables in the formula $(\forall x)(P(x) \rightarrow (\exists y)R(x, y))$ (Remembering)

11. Is the following argument valid? All lecturers are determined. Anyone who is determined and intelligent will give satisfactory service. Clare is an intelligent lecturer. There Clare will give satisfactory service. (use predicates). (Analyzing)

12. Prove that $(\exists x)M(x)$ follows logically from the premises $(x)(H(x) \rightarrow M(x))$ *and* $(\exists x)H(x)$ (Evaluating)

13. Prove that $(x)(P(x) \rightarrow Q(x)), (x)(R(x) \rightarrow \neg Q(x)) \Rightarrow (x)(R(x) \rightarrow \neg P(x))$ (Evaluating)

14. Prove that $(x)(H(x) \rightarrow A(x)) \Rightarrow (x)((\exists y)(H(y) \wedge N(x,y)) \rightarrow (\exists y)(A(y) \wedge N(x,y))$ (Evaluating)

UNIT – III: SET THEORY

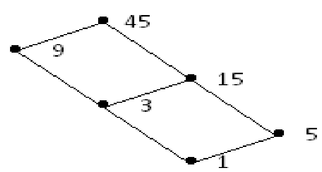
PART-A

1. If A = {2, 3} $\subseteq$ x {2, 3, 6, 12, 24, 36} and the relation ≤ is such that x ≤ y is x divides y, find the least element and greatest element for A. (Remembering)

Answer: There is no infimum. Supermum = 6.

2. Draw the Hasse diagram of (X, ≤),where X is the set of positive divisors of 45 and the relation ≤ is such that ≤ {(x,y : x ε A, y ε A ∧ (x divides y)}. (Applying)

Answer:



3.Give an example of a relation which is both symmetric and antisymmeteric. (Understanding)

Answer: Let R_2 be another relation on A i.e., $R_2 \subseteq A \times A$ given by $R_2 = \{(1,2),(2,1),(3,3),(2,3),(3,2)\}$ .For R_2 to be symmetric, if (x, y) ε R_2 then (y,x) should belong to R_2. Here (1, 2) ε R_2 and (2, 1) also belongs to R_2. Likewise (2, 3) and (3, 2) belong to R_2. There is no other pair (x, y) with x and y different. Hence R_2 is symmetric. But for R_2 to be antisymmeteric if (x, y) and (y, x) ε R_2, then x = y. Here (2, 3) and (3,2) ε R_2 but 2 and 3 are distinct elements of A. Hence R_2 is antisymmeteric. Thus R_2 is both symmetric and antisymmeteric.

4.Given S = {1, 2, 3,….10} and a relation R on S where R = {(x, y) / x+y = 10} what are the properties of the relation R? (Remembering)

Answer: R consists of pairs (x,y) such that x+y = 10 and x,y ε S. i.e., it consists of pairs (1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1). The properties are as follows:

(i)R is not reflexive as it does not contain pairs of the form (x,x),x ε S. i.e., it does not contain (1,1), (2,2),…,(10,10)

(ii) R is irreflexive, since x R x, for every x ε S. i.e., R does not contain any of the pairs (1, 1), (2, 2), ….,(10,10).

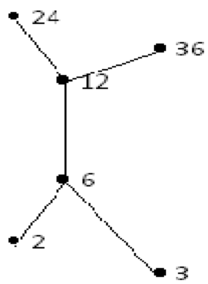
(iii) R is symmetric, as for any pair (x,y) in R, (y,x) also is in R. i.e., (1,9) and (9,1) ε R ; (2,8), (8,2) ε R and so on.

(iv) R is not anti symmetric for R contains elements (x, y) and (y, x) with x≠ y. For example, (1, 9), (9, 1) ε R but 1≠ 9.

(v) R is not transitive, for (1, 9) ε R and (9, 1) ε R but (1,1) does not belongs to R.

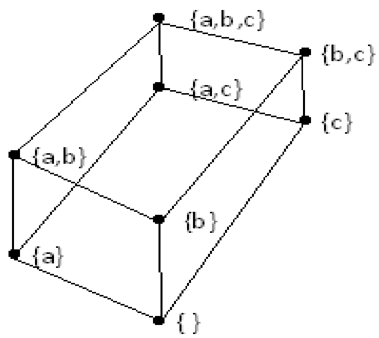
5. Let X = {2, 3, 6, 12, 24, 36} and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(x, \leq)$. (Applying)

Answer:



6.Obtain the Hasse diagram of $(P(A_3), \subseteq)$, where A_3 = {a, b, c}. (Evaluating)

Answer: Given S = {a,b,c} and P(S) = {{a},{b},{c},{a,b},{b,c},{a,c},{a,b,c},{}}. We know that $\{P(S), \subseteq\}$ is a poset. Since empty set is a subset of every set in P(S), { } is the least element of P(S). Similarly S = {a, b, c} contains all elements of P(S). i.e., an element of P(S) is a subset of {a, b, c}.Therefore S is the greatest element in P(S). $\therefore (P(S), \subseteq)$ is a lattice. The Hasse diagram is



7.Define partially ordered set. (Remembering)

Answer: The set with its partial order relation R is called POSET. (OR) A set P together with a partial ordering R is called a partially ordered set or a poset.

8. If a poset has a least element, then prove it is unique. (Evaluating)

Answer: Poset: $(L, \leq)$. a_1, a_2 are two least elements in L. Then $a_1 \leq a_2$, $a_2 \leq a_1$. By antisymmetry $a_2 = a_1$ so least element if exists is unique.

9. Give an example of a relation which is symmetric, transitive but not reflexive on {a, b, c}.

Answer: Let A = {a,b,c} and R : {(a,b),(b,a),(a,a),(b,b),(b,c),(a,c),(c,a),(c,b)}

1. Reflexive $\forall x \in A, (x, x) \in R$, $(c, c) \notin R$ $\because$ not reflexive. (Remembering)
2. Symmetry if $\forall$ $x, y \in A$, $x R y \Rightarrow y R x$
3. Transitive if $x R y$ and $y R z \Rightarrow x R z$ for all $x, y, z \in A$.

PART-B

1.Prove that distinct equivalence classes are disjoint. (Evaluating)

2. Let P={(1,2),(3,4),(5)} be a partition of the set S={1,2,3,4,5}. Construct an equivalence relation R on S so that the equivalence classes with respect to R are precisely the members of P. (Evaluating)

3. A survey of 500 television watchers produced the following information: 285 watch foot ball games ;195 watch hockey games; 115 watch Basket ball games; 45 watch foot ball and basket ball games; 70 watch foot ball and hockey games; 50 watch hockey and basket ball games; 50 do not watch any of the three games How many people watch exactly one of the three games? (Remembering)

4. Let the relation R be defined on the set of all real numbers by 'If x,y are real numbers, $xRy \Leftrightarrow x - y$ is a rational number'. Show that R is an equivalence relation. (Remembering)

5.Define the relation P on (1,2,3,4) by $P = \{(a,b) / |a-b| = 1\}$. Determine the adjacency matrix of P^2. (Remembering)

6. If r_1,r_2 are equivalence relations in a set A, then prove $r_1 \cap r_2$ is an equivalence relation in A.

7. If R is an equivalence relation on a set A, prove that [x] = [y] if and only if xRy where [x] and [y] denote equivalence classes containing x and y respectively. (Evaluating)

8. Show that the relation R={(x,y)/ x,y$\in$Z , x - y is divisible by 3} is an equivalence relation. (Remembering)

9. Draw the graph of the relation R= {(x,y)/x,y$\in$X, x>y } where X={1,2,3,4} and find the relation matrix. (Remembering)

UNIT – IV: FUNCTIONS

1. Show that the functions $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$, are inverse of one another. (Remembering)
   Answer: We use the condition for two functions to be inverses of each other. Here $f(x) = x^3$, $g(x) = x^{1/3}$ .Both f and g are functions from R to R.
   Then $(f \circ g)(x) = f(g(x)) = f(x^{1/3}) = (x^{1/3})^3 = x = I_x(x)$ and
   $(g \circ f)(x) = g(f(x)) = g(x^3) = (x^3)^{1/3} = x = I_x(x)$. Hence $g = f^{-1}$ or $f = g^{-1}$.

2. Let f: R → R and g: R → R where R is the set of real numbers find $f \circ g$ and $g \circ f$, if $f(x) = x^2 - 2$ and $g(x) = x+4$. (Remembering)
   Answer: For all $x \in R$, $g \circ f(x) = g(f(x)) = g(x^2 - 2) = x^2 - 2 + 4$ and
   $f \circ g(x) = f(g(x)) = f(x + 4) = (x + 4)^2 - 2 = x^2 + 8x + 14$.

3. The inverse of the inverse of a function is the function itself i.e., $(f^{-1})^{-1} = f$. (OR) If a function g be the inverse of a function f, then f is the inverse of g. (Evaluating)
   Answer: Let f: A → B be invertible. Then ∃ a function $g = f^{-1}$: B → A such that f(a) = b ⇒ a = g(b). Also f is invertible ⇒ f is one-one and onto. ⇒ g is one-one and onto. ⇒ g is invertible (i.e.,) g^{-1} exists.
   Now Then $(f \circ g)(b) = f[g(b)] = f(a) = b$. ⇒ $f \circ g = I_B$. ⇒ f is the inverse of g ⇒ $f = g^{-1}$.
   ⇒ $f = (f^{-1})^{-1}$.

4. Determine whether usual multiplication on the set A = {1,-1} is a binary operation.
   Answer: Since (-1).1 = -1 ∈A, 1.1 = 1 ∈A . (-1). (-1) ∈A, 1. (-1) = -1∈A usual multiplication on A is a binary opertation. (Understanding)

5. Examine whether matrix multiplication on the set. M = $\left\{\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} : a, b \in R \right\}$ is binary operation.

Answer: Let $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & c \\ d & 0 \end{bmatrix} \in M$

$\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix} \begin{bmatrix} 0 & c \\ d & 0 \end{bmatrix} = \begin{bmatrix} ad & 0 \\ 0 & bc \end{bmatrix} \notin M$. [This is not in the form of the element of M]

Therefore Matrix multiplication is not a binary operation on the set M. (Evaluating)

6. Is the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$ even or odd? (Remembering)

Answer: We first write p as a product of disjoint cycles, obtaining
p = (3, 5, 6) ° (1, 2, 4, 7).
Next we write of the cycles as a product of transpositions
(1, 2, 4, 7) = (1, 7) ° (1, 4) ° (1, 2) & (3, 5, 6) = (3, 6) ° (3, 5).
Then p = (3, 6) ° (3, 5) ° (1, 7) ° (1, 4) ° (1, 2). Since p is a product of an odd number of transpositions, it is an odd permutation.

7. Find all the mappings from A = {1, 2} to B = {3, 4}. (Remembering)

Answer: There are 4 mappings which can be easily found.

8. Let h(x, y) = g (f_1(x, y), f_2(x, y)) for all positive integers x and y, where
$f_1(x, y) = x^2 + y^2$, $f_2(x, y) = x$ and $g(x, y) = xy^2$. Find h(x, y) in terms of x and y. (Remembering)

Answer: $h(x, y) = g[f_1(x, y), f_2(x, y)] = g[x^2 + y^2, x] = (x^2 + y^2)[x^2] = (x^2 + y^2)(x^2)$ Since $g(x, y) = xy^2$.

9. If f : A → B and g: B → C are mappings and go f : A → C is one-to-one (Injection), prove that f is one-to-one .
(Evaluating)

Answer: If f(x) = f(y) then g[f(x)] = g[f(y)] ⇒ (g ° f) (x) = (g ° f) (y) ⇒ x = y since g ° f is one-to-one. Hence f is one-to-one.

10. Define characteristic function. (Remembering)

Answer: Let U be a universal set and S be an arbitrary subset of U. The function f_s: U→ [0, 1] defined by $f_s(x) = \left\{ \begin{array}{c} 1 \text{ if } x \in s \\ 0 \text{ if otherwise} \end{array} \right\}$ is called a characteristic function.

11. If A has m elements and B has n elements, how many functions are there from A to B?

Answer: To define a function f : A → B, for each a ∈ A, we have to select one element from B as the image of a. For a given a ∈ A, we have n choices viz., n elements of B. As there are m elements in A and for each element in A there are n choices, the number of such choices is n^m. Hence the number of distinct functions from A to B is n^m . (Remembering)

12. Show that x*y = x-y is not a binary operation over the set of natural numbers, but that it is a binary operation on the set of integers. Is it commutative or associative? (Remembering)

Answer: 2 * 3 = 2 – 3 = -1∉ N. Since * is not a binary operation over the set of natural numbers. But for 2 * 3 = 2 – 3 = -1∈ Z is a binary operation over the set of integers. (i) * is not commutative since
a * b ≠ b * a (i.e.,) a - b ≠ b - a (ii) * is not associative since a * (b * c) ≠ (a – b) – c.

UNIT – V: GRAPH THEORY AND COMBINATORICS

1. Define : Graph (Remembering)
   A graph (G) contains a set of vertices (V) and a set of edges(E).

2. Define : Simple Graph (Remembering)
   A directed or undirected graph which has neither self-loops nor parallel edges is called simple graph.

3. Define : Self Loop (Remembering)
   An edge which starts from a vertex and moves back to it is called a self-edge or self-loop.

4. Define : Pseudo-Graph (Remembering)
   A directed or undirected graph in which self-loop(s) and parallel edge(s) are allowed is called a Pseudo-Graph.

5. Define :Degree of vertex (Remembering)
   The degree of the vertex of a graph is the number of edges incident to the vertex, with loops counted twice. The degree of a vertex V is denoted by $\deg(V)$.

6. Define : Isolated vertex (Remembering)
   Let G be a graph and v be a vertex in G. We say that v is an isolated vertex if it is not incident with any edge.

7. Define : Null Graph (Remembering)
   A graph which contains only isolated nodes is called a null graph, i.e., the set of edges in a null graph is empty.

8. Define : Pendant vertex (Remembering)
   A vertex v of G is said to be a Pendant vertex if it has degree 1.

9. Define : Adjacent Vertices (Remembering)
   The edges E of an undirected graph G induce a symmetric binary relation $\sim$on V that is called the adjacency relation of G. Specifically, for each edge $\{u, v\}$ the vertices u and v are said to be adjacent to one another, which is denoted u$\sim$v.

10. Define : Complete Graph (Remembering)
    A simple graph G is said to be complete if every vertex of G is connected with every other vertex of G , i.e., every pair of distinct vertices contains exactly one edge.

11. Define : Regular Graph (Remembering)
    A graph in which all the vertices are of same degree is called a regular graph.

12. Define : Bipartite Graph (Remembering)
    A graph $G = (V, E)$ is a bipartite Graph if the vertex set Vcan be partitioned into two disjoint subsets, say, V_1 and V_2 such that every edge in E connects a vertex in V_1 and a vertex V_2 is called a bipartition of G.

13. Define : Complete Bipartite Graph (Remembering)
A bipartite Graph G is a complete bipartite Graph if there is an edge between every pair of vertices taken from two disjoint sets of vertices.

14. Define: Isomorphic Graph (Remembering)
Two graphs $G = (V, E)$ and $G' = (V', E')$ are said to be isomorphic, if there exists a bi-jection $f: V \to V'$ such that $(U, V) \in E$ if and only if $\big(f(u), f(v)\big) \in E'$.

15. Define : Connected Graph (Remembering)
A graph is called connected it and only if for any pair of nodes u, v, there is at least one path between u and v. otherwise, it is disconnected.

16. Define : Euler Graph (Remembering)
A graph G=(V,E) is a Euler Graph if it has at least one Euler circuit.

17. Define : Euler path and Euler circuit (Remembering)
An Euler path consists of every edge of G exactly once. To find such a path, vertices may be repeated. In particular, if the first and the last traversed vertices are same on an Euler path, then that path is called as Euler circuit.

18. Define :Planar Graph (Remembering)
A graph said to be planar if it can be drawn on a plane so that no two edges intersect.

19. Properties of planar graph (Remembering)
   (a) If a connected simple planar graph G has e edges and r regions, then $r \leq \frac{2e}{3} r$
   (b) If a connected planar graph G has e edges, n vertices, and r regions, then $n - e + r = 2$ [Euler's rule].
   (c) If a connected simple planar graph G has $e$edges and $n \geq 3$vertices, then$e \leq 3n - 6$ e.
   (d) If a connected simple planar graph G has e edges and $n \geq 3$ vertices and no circuits of length 3, then $e \leq 2n - 4$.
   (e) A complete graph K_n is planar if $n < 5$
   (f) A complete bipartite graph $K_{m,n}$ is planar if and only if $m < 3$ or$n < 3$.

20. Define: chromatic number (Remembering)
The minimum number of colors needed to produce a proper coloring of a graph G is called the chromatic number of G

21. Define: Independent set (Remembering)
An independent set or stable set is a set of vertices in a graph, no two of which are adjacent.

22. Define: Dominating set (Remembering)
A dominating set for agraph$G = (V, E)$ is a subset D of V such that every vertex not in D is joined to at least one member of $D$by some edge. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G.

23.Define: chromatic polynomial (Remembering)
The chromatic polynomial of a graph counts the number of its proper vertex coloring. It is commonly denoted $P_G(k), \chi_G(k)$ or $\pi_G(k)$ and sometimes in the form$P(G; k)$, where it is understood that for fixed G the function is a polynomial ink, the number of colors.

PART-B

1. A simple graph with $n \geq 2$ vertices contains at least two vertices of same degree. (Remembering)
2. Show that the two graphs are isomorphic(Remembering)
3. Are the graph G' and G'' are isomorphic? (Remembering)
4. Prove that every complete graph K_n has $\frac{(n-1)!}{2}$ possible Hamiltonian circuits. (Evaluating)
5. Is graph shown as a Hamiltonian graph? (Remembering)
6. The sum of the degrees of the points of a graph G is twice the number of lines. (Evaluating)
7. In any graph G the number of points of odd degree is even. (Evaluating)
8. Every cubic graph has an even number of points. (Evaluating)
9. A closed walk of odd length contains a cycle. (Evaluating)
10. If G is not connected then $\overline{G}$ is connected. (Evaluating)
11. The following statement are equivalent for a connected graph G.
    (i) G is Eulerian, (ii) Every point of a G has even degree. (Evaluating)
    (iii) The set of edges of G can be partitioned into cycles.
12. A complete graph G is an Euler graph iff it can be decomposed into circuits. (Evaluating)
13. The complete graph of fine vertices is nonplanar. (Evaluating)
14. A connected planar graph with nvertices and e edges has $e - n + 2$ regions. (Evaluating)
15. State the properties of planar graph. (Remembering)
16. Prove that complete graph K_4 is planar. (Evaluating)
17. Prove that complete graph K_5 is not planar. (Evaluating)
18. Show that the graph $K_{3,3}$ is not planar. (Remembering)

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