

**K.S.R. COLLEGE OF ENGINEERING (Autonomous)****Vision of the Institution**

- We envision to achieve status as an excellent educational institution in the global knowledge hub, making self-learners, experts, ethical and responsible engineers, technologists, scientists, managers, administrators and entrepreneurs who will significantly contribute to research and environment friendly sustainable growth of the nation and the world.

**Mission of the Institution**

- To inculcate in the students self-learning abilities that enable them to become competitive and considerate engineers, technologists, scientists, managers, administrators and entrepreneurs by diligently imparting the best of education, nurturing environmental and social needs.
- To foster and maintain a mutually beneficial partnership with global industries and Institutions through knowledge sharing, collaborative research and innovation.

**DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING****Vision of the Department**

- To create ever green professionals for software industry, academicians for knowledge cultivation and researchers for contemporary society modernization.

**Mission of the Department**

- To produce proficient design, code and system engineers for software development.
- To keep updated contemporary technology and fore coming challenges for welfare of the society.

**Programme Educational Objectives (PEOs)**

**PEO1** : Figure out, formulate, analyze typical problems and develop effective solutions by imparting the idea and principles of science, mathematics, engineering fundamentals and computing.

**PEO2** : Competent professionally and successful in their chosen career through life-long learning.

**PEO3** : Excel individually or as member of a team in carrying out projects and exhibit social needs and follow professional ethics.

**K.S.R. COLLEGE OF ENGINEERING (Autonomous)**  
**Department of Computer Science and Engineering**

**Subject Name: NUMERICAL COMPUTATIONAL TECHNIQUES**

**Subject Code: 18MA331**

**Year/Semester: II/ IV**

*Course Outcomes: On completion of this course, the student will be able to*

- CO1* Enable to solve polynomial, transcendental equations and simultaneous linear equations numerically.
- CO2* To know the basics of Interpolation techniques.
- CO3* Develop their skills in numerical differentiation and integration.
- CO4* Finding numerical solutions to ordinary differential equations.
- CO5* Able to apply the concepts of numerical solutions to boundary values problems

**Program Outcomes (POs) and Program Specific Outcomes (PSOs)**

**A. Program Outcomes (POs)**

**Engineering Graduates will be able to :**

- Engineering knowledge:** Ability to exhibit the knowledge of mathematics, science, engineering fundamentals and programming skills to solve problems in computer science.
- PO1** fundamentals and programming skills to solve problems in computer science.
- PO2 Problem analysis:** Talent to identify, formulate, analyze and solve complex engineering problems with the knowledge of computer science. .
- PO3 Design/development of solutions:** Capability to design, implement, and evaluate a computer based system, process, component or program to meet desired needs.
- PO4 Conduct investigations of complex problems:** Potential to conduct investigation of complex problems by methods that include appropriate experiments, analysis and synthesis of information in order to reach valid conclusions.
- PO5 Modern tool Usage:** Ability to create, select, and apply appropriate techniques, resources and modern engineering tools to solve complex engineering problems.
- PO6 The engineer and society:** Skill to acquire the broad education necessary to understand the impact of engineering solutions on a global economic, environmental, social, political, ethical, health and safety.
- PO7 Environmental and sustainability:** Ability to understand the impact of the professional engineering solutions in societal and Environmental contexts and demonstrate the knowledge of, and need for sustainable development.
- PO8 Ethics:** Apply ethical principles and commit to professional ethics and responsibility and norms of the engineering practices.
- PO9 Individual and team work:** Ability to function individually as well as on multi-disciplinary teams.
- PO10 Communication:** Ability to communicate effectively in both verbal and written mode to excel in the career.
- PO11 Project management and finance:** Ability to integrate the knowledge of engineering and management principles to work as a member and leader in a team on diverse projects.
- PO12 Life-long learning:** Ability to recognize the need of technological change by independent and life-long learning.

**B. Program Specific Outcomes (PSOs)**

- PSO1** Develop and Implement computer solutions that accomplish goals to the industry, government or research by exploring new technologies.
- PSO2** Grow intellectually and professionally in the chosen field.

**UNIT – I : SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS****PART-A**

1. State the order of convergence and convergence condition for Newton's Raphson method. **(Understanding)**

Order of convergence is two. Convergence condition is  $|f(x) f'(x)| < |f'(x)|^2$ .

2. Write the iterative formula for Newton's Raphson method. **(Remembering)**

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0, 1, 2, \dots$$

3. Newton's Raphson method is otherwise known as -----.(Method of tangents) **(Understanding)**

4. Find an iterative formula to find  $\sqrt{N}$ , where N is a positive number. **(Evaluating)**

$$x_{n+1} = \frac{x_n^2 + N}{2x_n}$$

5. What is the Newton Raphson formula for cube root of N? **(Remembering)**

$$x = \sqrt[3]{N}, x^3 = N, f(x) = x^3 - N, x_{n+1} = \frac{1}{3} \left( x_n + \frac{N}{x_n^2} \right)$$

6. In Newton Raphson method the error at any stage is proportional to the \_\_\_\_\_ of the error in the previous stage. (Square) **(Understanding)**

7. The numerical methods of solving linear equations are of two types: one is direct and the other is ----- . (Iterative) **(Understanding)**

8. If  $f(x)$  is continuous in  $(a,b)$  and if  $f(a).f(b) < 0$  then  $f(x) = 0$  will have at least \_\_\_\_\_ **(Understanding)**

(One real root between 'a' and 'b')

9. If  $f(x) = 0$  has root between  $x = a$  and  $x = b$ , then write the first approximate root by the method of false position. **(Remembering)**

$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

10. Compare Gauss elimination and Gauss –Seidel methods for solving a linear systems **(Analyzing)**

Gauss elimination method is direct method. Gauss –Seidel method is iterative method.

11. State the condition for the convergence of Gauss –Seidel method. **(Understanding)**

Coefficient matrix should be diagonally dominant.

12. For solving a linear system, compare Gaussian elimination method and Gauss – Jordan method. **(Analyzing)**

In Gaussian elimination method the given system is transformed into an equivalent system with upper triangular coefficient matrix and then using back substitution we can find the unknowns. In Gauss – Jordan method the coefficient matrix is reduced to an unit matrix and then directly we can find the unknowns.

13. State the principle used in Gauss – Jordan method. **(Understanding)**

In Gauss – Jordan method the coefficient matrix is reduced to an unit matrix and then directly we can find the unknowns.

14. Solve the system of equations  $x - 2y = 0, 2x + y = 5$  by Gauss elimination method. **(Evaluating)**

$$[A, B] = \begin{pmatrix} 1 & -2 & 0 \\ 2 & 1 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 5 & 5 \end{pmatrix}, x = 2, y = 1.$$

15. When Gauss – Jordan method is used to solve  $AX = B$ , A is transferred in to a ----- matrix. **(Analyzing)**

Unit matrix.

16. What type of Eigen value can be obtained using power method? **(Remembering)**

Dominant Eigen value of the given matrix.

17. Find the dominant eigen value of  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  by power method. **(Evaluating)**

Dominant eigen value is 5.3722

18. State the two differences between direct and iterative methods for solving system of equations. **(Understanding)**

Direct method	Iterative method
It gives exact value	It gives only approximate solution
Take less time and simple	Time consuming and laborious

### **PART-B**

- Find the real positive root of  $3x - \cos x - 1 = 0$  by Newton's Method correct to 3 decimal places. **(Applying)**
- Find a root of  $f(x) = xe^x - \cos x$  lying in the interval (0, 1) by Newton Raphson method. **(Applying)**
- Find an iterative formula to find the reciprocal of given number N by Newton Raphson method and hence find the value of  $1/19$ . **(Applying)**
- Using Newton Raphson method, Solve  $x \log_{10} x = 12.34$ . Start  $x_0 = 10$ . **(Analyzing)**
- Find a root of  $x \log_{10} x - 1.2 = 0$  by False position method correct to 4 decimal places. **(Applying)**
- Find a root of  $x \log_{10} x - 1.2 = 0$  by Newton's method correct to three decimal places. **(Applying)**
- Solve by Gauss Elimination Method  $3x + 4y + 5z = 18, 2x - y + 8z = 13, 5x - 2y + 7z = 20$ . **(Evaluating)**
- Solve  $10x + y + z = 12, 2x + 10y + z = 13, x + y + 5z = 7$  by Gauss Jordan method. **(Evaluating)**

9. Using Gauss Jordan method, find the inverse of  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 1 & 2 & 1 \end{bmatrix}$  **(Applying)**

10. Using Gauss Jordan method, find the inverse of  $A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$  **(Applying)**

11. Find, by Gauss, Jordan method, the inverse of the matrix  $A = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$ .

**(Evaluating)**

12. Using Gauss Jordan method, Find the inverse of the matrix  $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$  **(Applying)**

13. Find the inverse of the matrix by Gauss Jordan method  $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$  **(Applying)**

14. Solve the given system of equations by using Gauss – Seidal iteration method.

$$27x + 6y - z = 85, 6x + 15y + 2z = 7, x + y + 54z = 110. \text{ (Evaluating)}$$

15. Solve the equations By Gauss Seidal method ( 5 Iterations are enough)

$$30x - 2y + 3z = 75, 2x + 2y + 18z = 30, x + 17y - 2z = 48. \text{ (Evaluating)}$$

16. Solve the given system of equations by using Gauss – Seidal method iteration method  $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25. \text{ (Evaluating)}$

17. Solve the given system of equations by using Gauss – Seidal method iteration method  $10x + 2y + z = 9, x + 10y - z = -22, -2x + 3y + 10z = 22. \text{ (Evaluating)}$

18. Determine the largest eigen value and corresponding eigen vector of the matrix by

power method  $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$  **(Evaluating)**

19. Find all eigen values of the matrix by power method **(Applying)**

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

20. Find the numerically dominant eigen value and the corresponding eigen vector by

power method if  $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 5 \end{bmatrix}$  **(Applying)**

21. Find the largest eigen value and the corresponding eigen vector by power method if

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ (Applying)}$$

22. Find the largest eigen values and eigen vectors of the matrix by power method

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix} \text{ (Applying)}$$

## **UNIT – II : INTERPOLATION AND APPROXIMATION**

### **PART-A**

1. Define Interpolation and Extrapolation? **(Understanding)**

The process of finding the value of a function inside the given range is called Interpolation.

The process of finding the value of a function outside the given range is called Extrapolation.

2. Why the polynomial interpolation is preferred mostly? **(Analyzing)**

a. They are simple forms of functions which can be easily manipulated.

- b. Computations for definite values of the argument, integration and differentiation of such Functions are easy.
- c. Polynomials are free from singularities where as rational functions or other types, do have Singularities.
3. Write the Newton's forward interpolation formula. Give the meaning of first two terms and first three terms of it. **(Remembering)**

$$Y(x) = y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \dots + \frac{u(u-1)\dots(u-(n-1))}{n!} \Delta^n y_0,$$

$$\text{where } u = \frac{x - x_0}{h}.$$

In the above formula, the first two terms will give the linear interpolation and the first three terms will give a parabolic interpolation.

4. Write Newton's Backward Interpolation formula. **(Remembering)**

$$Y(x) = y_n + \frac{v}{1!} \nabla y_n + \frac{v(v+1)}{2!} \nabla^2 y_n + \dots + \frac{v(v+1)\dots(v+(n-1))}{n!} \nabla^n y_n,$$

$$\text{where } v = \frac{x - x_n}{h}$$

5. State Newton's divided difference formula. **(Remembering)**  
 $f(x) = f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + \dots + (x-x_0)(x-x_1)\dots(x-x_{n-1})f(x_0, x_1, x_2, \dots, x_n).$
6. State the error in Newton's forward interpolation and Newton's backward interpolation formula. **(Remembering)**

Error in forward interpolation =

$$f(x) - p_n(x) = \frac{u(u-1)(u-2)\dots(u-n)}{(n+1)!} h^{n+1} f^{(n+1)}(c), \text{ where } u = \frac{x - x_0}{h}.$$

Error in backward interpolation =

$$f(x) - p_n(x) = \frac{v(v+1)(v+2)\dots(v+n)}{(n+1)!} h^{n+1} f^{(n+1)}(c), \text{ where } v = \frac{x - x_n}{h}.$$

7. Show that  $\Delta_{bcd}^3 \left( \frac{1}{a} \right) = -\frac{1}{abcd}$ . **(Applying)**
8. Fill in the blanks:-The nth order difference of the polynomial  $p = a_0x^n + a_1x^{n-1} + \dots + a_n$  is Constant. **(Understanding)**
9. Newton's divided difference formula for equal interval is called Newton's Gregory forward difference formula. **(Remembering)**
10. State True or False:-"Lagrange's interpolation formula can be used whether the arguments are equally spaced or not". **(Analyzing)** Ans : True
11. State True or False:-"The third differences of a polynomial of degree 4 are zeros". **(Analyzing)**  
 Ans : False
12. State True or False:-"The nth divided differences of a polynomial of nth degree are not constant". **(Analyzing)** Ans : True.
13. If  $f(x) = 1/x$ , find the divided differences  $f(a,b)$  and  $f(a,b,c)$ . **(Evaluating)**

14. State True or False: “The nth differences of a polynomial of degree n are zeros”.

**(Analyzing)**

Ans : True

15. State Lagrange's Interpolation formula. **(Remembering)**

Let  $y = f(x)$  be a function such that  $f(x)$  takes the values  $y_0, y_1, \dots, y_n$  corresponding to  $x_0, x_1, \dots, x_n$ , then  $y = f(x) =$

$$\frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n$$

16. State inverse Lagrange's Interpolation formula. **(Remembering)**

$X =$

$$\frac{(y-y_1)(y-y_2)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)\dots(y_0-y_n)} x_0 + \frac{(y-y_0)(y-y_2)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)\dots(y_1-y_n)} x_1 + \dots + \frac{(y-y_0)(y-y_1)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)\dots(y_n-y_{n-1})} x_n$$

17. When will you use Newton's backward interpolation formula? **(Applying)**

We can apply the Newton's backward interpolation if the unknown value lies near the end of the table value.

18. Using Lagrange's interpolation formula, find the polynomial for **(Applying)**

x	0	1	3	4
y	-12	0	0	12

$y = f(x) =$

$$\frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots(x_n-x_{n-1})} y_n \text{ (i.e.)}$$

$$y = \frac{(x-1)(x-3)(x-4)}{(0-1)(0-3)\dots(0-4)} (-12) + \frac{(x)(x-3)(x-4)}{(1-0)(1-2)(-4)} 0 + \frac{x(x-1)(x-4)}{(3-0)(3-1)(3-4)} 0 + \frac{x(x-1)(x-3)}{(4-0)(4-1)(4-3)} x$$

$$\text{(i.e.) } y(x) = 2x^3 - 12x^2 + 22x - 12.$$

19. Obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula. **(Applying)**

x	0	2	4	6
y	-3	5	21	45

Solution: The finite difference table is

X	Y	$\Delta y$	$\Delta^2 y$	$\Delta^3 y$
0	-3			
2	5	8		
4	21	16	8	
6	45	24	8	0

$$u = \frac{x-0}{2} = \frac{x}{2}. \text{ Therefore } f(x) = y_0 + u \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 = x^2 + 2x - 3.$$

20. Find the parabola of the form  $y = ax^2 + bx + c$  passing through the points (0, 0) (1, 1) and (2, 20). **(Evaluating)**

$$\begin{aligned} \text{By Lagrange's formula } y &= \frac{(x-1)(x-2)}{(0-1)(0-2)} (0) + \frac{(x-0)(x-2)}{(1-0)(1-2)} 1 + \frac{(x-0)(x-1)}{(2-0)(2-1)} 20 \\ &= 9x^2 - 8x. \end{aligned}$$

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21. State any two properties of divided difference. **(Remembering)**

- (i) The divided differences are symmetrical in all their arguments.
- (ii) The operator  $\Delta$  is linear.
- (iii) nth divided differences of a polynomial of degree 'n' are constants.

22. State True or False.

“Lagrange’s interpolation formula is applicable for both equal and unequal intervals”.

**(Analyzing)** (Ans: True).

23. Fill in the blank: If the tabulated function is a polynomial of degree  $n$ , then  $\Delta^n y_o$  is a constant. **(Remembering)**

(or)  $n^{\text{th}}$  divided differences of a polynomial of degree  $n$  is constant.

24. Construct a linear interpolating polynomial given the points  $(x_o, y_o)$  and  $(x_1, y_1)$ . **(Creating)**

$$\text{Let } y = ax + b$$

$$Y_o = ax_o + b$$

$$Y_1 = ax_1 + b.$$

$$\text{Elimination of } a \text{ and } b \Rightarrow \begin{vmatrix} y & -x & -1 \\ y_o & -x_o & -1 \\ y_1 & -x_1 & -1 \end{vmatrix} = 0 \Rightarrow y = y_o \left[ \frac{x - x_1}{x_o - x_1} \right] + y_1 \left[ \frac{x - x_o}{x_1 - x_o} \right].$$

25. Find the divided difference of  $f(x) = x^3 - 2x$  for the arguments 2, 4, 9, 10. **(Evaluating)**

$$\frac{1}{x^2}$$

26. If  $f(x) = \frac{1}{x^2}$  then the divided difference  $f(a, b)$  is  $\frac{-(a+b)}{a^2b^2}$ . **(Understanding)**

27. Write the divided difference table for the following. **(Creating)**

X:	1	2	4	7
Y:	22	30	82	106

28. Show that the divided differences are symmetrical in their arguments. **(Analyzing)**

$$f(x_o, x_1) = \frac{f(x_1) - f(x_o)}{x_1 - x_o} = \frac{f(x_o) - f(x_1)}{x_o - x_1} = f(x_1, x_o).$$

Similarly we can prove that  $f(x_1, x_2) = f(x_2, x_1)$ .

29. Obtain the divided difference table for the following data. **(Understanding)**

X:	5	7	11	13	17
Y:	150	392	1452	2366	5202

30. Define forward, backward and divided difference. **(Remembering)**

$$\Delta f(x) = f(x+h) - f(x) \quad \nabla f(x) = f(x) - f(x-h)$$

$$\Delta f(x) = \frac{f(x_1) - f(x_o)}{x_1 - x_o}.$$

31. Show that the divided difference operator  $\Delta$  is linear. **(Analyzing)**

Let  $\alpha$  and  $\beta$  are two constants,  $f(x)$  and  $g(x)$  are two functions.

$$\Delta[\alpha f(x) + \beta g(x)] = \frac{[\alpha f(x_1) + \beta g(x_1)] - [\alpha f(x_o) + \beta g(x_o)]}{x_1 - x_o}$$

$$= \alpha \cdot \frac{f(x_1) - f(x_o)}{x_1 - x_o} + \beta \frac{g(x_1) - g(x_o)}{x_1 - x_o} = \alpha \Delta f(x) + \beta \Delta g(x).$$

Therefore  $\Delta[\alpha f(x) + \beta g(x)] = \alpha \Delta f(x) + \beta \Delta g(x)$ . Hence  $\Delta$  is linear.

32. Find the missing value of the table given below. What assumption have you made to find it? **(Evaluating)**

Year	:	1917	1918	1919	1920	1921
Export (in tons) :		443	384	--	397	467



Soln.: Since four values are given, we assume that we have a third degree polynomial and hence its fourth order differences of  $P_3(x)$  are zeros.

Let  $u_0 = 443$ ,  $u_1 = 384$ ,  $u_2 = ?$ ,  $u_3 = 397$ ,  $u_4 = 467$ .

Clearly  $\Delta^4 u_0 = 0 \Rightarrow (E - 1)^4 u_0 = 0 \Rightarrow (E^4 - 4E^3 + 6E^2 - 4E + 1)u_0 = 0$

$\Rightarrow u_4 - 4u_3 + 6u_2 - 4u_1 + u_0 = 0 \Rightarrow 467 - 4(397) + 6u_2 - 4(384) + 443 = 0$ .

$$\Rightarrow 6u_2 = 2214 \Rightarrow u_2 = 369.$$

### PART-B

1. For the given values evaluate  $f(9)$  using Lagrange's formula (**Evaluating**)

x	5	7	11	13	17
y	150	392	1452	2366	5202

2. Find the value of y at  $x = 6$  by Newton's divided difference formula for the data: (**Evaluating**)

x	-1	0	2	3	7	10
y	-11	1	1	1	141	561

3. Find  $f(0.47)$  by using Newton's Backward difference formula for (**Evaluating**)

x	0	0.1	0.2	0.3	0.4	0.5
f(x)	1	1.1103	1.2428	1.3997	1.5836	1.7974

4. Using Lagrange's formula, find  $f(323.5)$  for the data (**Analyzing**)

x	321.0	322.8	324.2	325.0
f(x)	2.50651	2.50893	2.51081	2.51188

5. Find the value of y at  $x = 1.05$  from the following table given below: (**Evaluating**)

x	1.0	1.1	1.2	1.3	1.4	1.5
y	0.841	0.891	0.932	0.964	1.985	1.015

6. Using Lagrange's interpolation formula, find  $f(6)$  from the following data: (**Analyzing**)

x	2	5	7	10	12
y	18	180	448	1210	2028

7. By using Newton's divided difference formula find  $f(-2)$  and  $f(12)$  from the following data: (**Analyzing**)

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

8. From the data given below, find the value of x when  $y = 13.5$  by Lagrange's inverse interpolation. (**Evaluating**)

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

9. From the following table, evaluate  $f(3.8)$  using Newton backward interpolation formula. (**Evaluating**)

x	0	1	2	3	4
f(x)	1.00	1.50	2.20	3.10	4.60

10. Use Lagrange's formula to calculate  $f(3)$  from the following table. (**Applying**)

x	0	1	2	4	5	6
y	1	14	15	5	6	1

11. Given

x	1	2	3	4	5	6	7	8
y	1	8	27	64	125	216	343	512

Estimate  $f(7.5)$ . Use Newton's formula. **(Evaluating)**

12. Using Newton's divided difference formula, find  $f(x)$  from the following data and hence find  $f(4)$  **(Applying)**

x:	0	1	2	5
f(x):	2	3	12	147

13. Find the value of  $y$  when  $x = 5$  using Newton's interpolation formula from the following table. **(Evaluating)**

x:	4	6	8	10
y :	1	3	8	16

14. Use Newton's divided difference formula to find  $f(x)$  from the following data **(Applying)**

x :	1	2	7	8
y:	1	5	5	4

15. Apply Lagrange's formula to find  $y(27)$  to the data given below. **(Applying)**

x:	14	17	31	35
y:	68.8	64	44	39.1

23. Fit a polynomial, by using Newton's forward interpolation formula, to the data given below. **(Applying)**

x :	0	1	2	3
y :	1	2	1	10

### **UNIT III – NUMERICAL DIFFERENTIATION AND INTEGRATION**

1. What are the errors in Trapezoidal and Simpson's rules of numerical integration? **(Understanding)**

The error in the Trapezoidal is  $E < \frac{(b-a)h^2}{12} y''(\xi)$

The error in Simpson's rule is  $E < \frac{-h^4}{180} (b-a)$

2. Using Newton's backward difference, write the formulae for the first and second order derivatives at  $x = x_n$  **(Understanding)**

$$\left( \frac{dy}{dx} \right)_{x=x_n} = \frac{1}{h} \left( \nabla y_n + \frac{1}{2} \nabla^2 y_n + \frac{1}{3} \nabla^3 y_n + \dots \right)$$

$$\left( \frac{d^2 y}{dx^2} \right)_{x=x_n} = \frac{1}{h^2} \left( \nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \dots \right)$$

3. In order to evaluate by Simpson's  $\frac{1}{3}$  rule as well as by Simpson's  $\frac{3}{8}$  rule, what is the restriction on the number of intervals? **(Evaluating)**

In Simpson's  $\frac{1}{3}$  rule the number of sub intervals should be even. In Simpson's  $\frac{3}{8}$  rule the number of sub intervals should be a multiple of 3.

4. Write the Simpson's  $\frac{3}{8}$ th formula. **(Remembering)**

$$\int_{x_0}^{x_0+nh} f(x)dx = \frac{3h}{8} \{ (y_0 + y_n) + 3(y_1 + y_2 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3}) \}.$$

5. Write down the expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $x = x_0$  by Newton's forward difference formula. **(Remembering)**

$$\left( \frac{dy}{dx} \right)_{x=x_0} = \frac{1}{h} \left( \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \dots \right)$$

$$\left( \frac{d^2y}{dx^2} \right)_{x=x_0} = \frac{1}{h^2} \left( \Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_0 - \dots \right)$$

6. Evaluate  $\int_1^4 f(x)dx$  from the table by Simpson's  $\frac{3}{8}$  rule. **(Evaluating)**

x	1	2	3	4
f(x)	1	8	27	64

By Simpson's  $\frac{3}{8}$  rule,  $\int_1^4 f(x)dx = \frac{3}{8} ((1 + 64) + 3(8 + 27))$

7. Using Trapezoidal rule evaluate  $\int_0^{\pi} \sin x dx$  by dividing the range into 6 equal parts.

**(Applying)**

Ans : 1.97

8. State Trapezoidal rule. **(Remembering)**

$$\int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2} ((y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})).$$

9. **Error in Trapezoidal rule is of order -----, ( $h^2$ ) (Remembering)**  
 10. **Error in Simpson's rule is of order-----, ( $h^4$ ) (Remembering)**  
 11. **A curve passes through (0,1),(0.25,0.9412),(0.5 , 0.8),(0.75,0.64) and (1,0.5).Find the area between the curve x- axis and x = 0 and 1 by trapezoidal rule.(Understanding)**  
 Ans :0.7828  
 12. **Which one is more reliable Simpson's rule or Trapezoidal rule?(Analyzing)**  
 Simpson's rule.

### **PART-B**

1. The table below reveals the velocity 'v' of a body during the time 't' specified.

Find its acceleration at  $t = 1.1$ . **(Understanding)**

t	1	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60

2. Compute  $f'(x)$  and  $f''(x)$  at  $x = 25$  from the following table. **(Evaluating)**

x	15	17	19	21	23	25
f(x)	3.873	4.123	4.359	4.583	4.796	5.8

3. Find the First and Second order derivatives of  $x^{1/3}$  at  $x = 50$  and  $x = 56$ . **(Evaluating)**

x	50	51	52	53	54	55	56
$y = x^{1/3}$	3.684	3.7084	3.7325	3.7563	3.7798	3.803	3.8259

4. Evaluate  $\int_0^1 \int_1^2 \frac{2xy}{(1+x^2)(1+y^2)} dx dy$  by Trapezoidal rule and Simpson's rule with taking  $h = k = 0.25$ . **(Evaluating)**
5. Evaluate  $\int_1^2 \int_1^2 \left( \frac{dx dy}{x+y} \right)$  by using Trapezoidal rule with  $h = k = 0.25$ . **(Evaluating)**
6. Solve  $\int_1^{1.4} \int_2^{2.4} \frac{1}{xy} dx dy$  using Simpson's rule by taking  $h = k = 0.1$ . **(Applying)**
7. Find the numerical value of the first derivative at  $x = 0.4$  of the function  $f(x)$  defined as under
- |      |         |        |         |         |
|------|---------|--------|---------|---------|
| x    | 0.1     | 0.2    | 0.3     | 0.4     |
| f(x) | 1.10517 | 1.2214 | 1.34986 | 1.49182 |
- (Evaluating)**
8. Evaluate  $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$  by using Simpson's rule. **(Evaluating)**
9. Use Simpson's  $1/3^{\text{rd}}$  rule to estimate the value of  $\int_1^5 f(x) dx$ . Given **(Applying)**
- |      |    |    |    |    |     |
|------|----|----|----|----|-----|
| x    | 1  | 2  | 3  | 4  | 5   |
| f(x) | 13 | 50 | 70 | 80 | 100 |
10. Compute  $\int_0^{\pi/2} \sin x dx$  using Simpson's  $\frac{3}{8}$  rule. **(Applying)**
11. Evaluate  $\int_0^2 \int_0^1 4xy dx dy$  using Simpson's rule by taking  $h = \frac{1}{4}$  and  $k = \frac{1}{2}$ . **(Evaluating)**
12. Using Trapezoidal rule, evaluate  $\int_{-1}^1 \frac{1}{1+x^2} dx$  by taking eight equal intervals. **(Applying)**

## UNIT IV-INITIAL VALUE PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS

### PART-A

1. Given  $y' = y - x^2$ ,  $y(0) = 1$ , find  $y(0.1)$  by Taylor's method. **(Understanding)**  
Ans: 1.10482
2. Write the algorithm for Euler's method. **(Remembering)**

Ans:  $y_{n+1} = y_n + hf(x_n, y_n)$

3. What are limitations of Euler's method? **(Remembering)**

Ans: 1.The attainable accuracy is limited by length of step h.

2.The method is slow and limited accuracy.

4. Error in Modified Euler's method is \_\_\_\_\_ **(Remembering)**

Ans:  $\text{Error} = \frac{-h^3}{12} \times \text{const} \tan t = O(h^3)$

5. Given  $y' = y - x^2$ ,  $y(0) = 1$ , find  $y(0.2)$  by Modified Euler's method. **(Applying)**

Ans: 1.218

6. Write the first order Runge Kutta method. **(Remembering)**

Ans:  $y_1 = y_0 + h(x_0, y_0), \quad y_1 = y_0 + k_1$

7. Write the fourth order Runge Kutta method. **(Remembering)**

Ans:  $k_1 = hf(x_n, y_n), \quad k_2 = hf(x_n + h/2, y_n + k_1/2)$

$k_3 = hf(x_n + h/2, y_n + k_2/2), \quad k_4 = hf(x_n + h, y_n + k_3)$

$\Delta y_n = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \quad y_{n+1} = y_n + \Delta y_n, \quad n = 0, 1, 2, 3, \dots$

8. Given  $y' = 1 - y$ ,  $y(0) = 0$  find  $y(0.1)$  by R.K method. **(Applying)**

Ans:  $y(0.1) = 0.1$

9. What do you meant by single step methods Give Example. **(Remembering)**

Ans: Taylor's, Euler's, Modified Euler's, Runge Kutta .... etc.

10. What do you meant by multi step methods Give Example. **(Remembering)**

Ans: Milne's, Adam's.

11. Runge Kutta method of First order is nothing but \_\_\_\_\_ **(Remembering)**

Ans: Euler's method.

12. Runge Kutta method of second order is nothing but \_\_\_\_\_ **(Remembering)**

Ans: Modified Euler's method.

13. Write down Milnes Corrector formula. **(Understanding)**

Ans:  $y_{n+1,C} = y_{n-1} + \frac{h}{3}(2y_{n-1}' + 4y_n' + y_{n+1}')$

14. Write down Milnes predictor formula. **(Understanding)**

Ans:  $y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y_{n-2}' - y_{n-1}' + 2y_n')$

15. Write down Adam's Corrector formula. **(Understanding)**

Ans:  $y_{n+1,C} = y_n + \frac{h}{24}(9y_{n+1}' + 19y_n' - 5y_{n-1}' + y_{n-2}')$

16. Write down Adam's predictor formula. **(Understanding)**

Ans:  $y_{n+1,p} = y_n + \frac{h}{24}(55y_n' - 59y_{n-1}' + 37y_{n-2}' - 9y_{n-3}')$

17. Given  $y' = f(x, y)$ ,  $y(x_n) = y_n$ , write down Taylor's series for  $y^{n+1}$ . **(Understanding)**

$$\text{Ans: } y_{n+1} = y_n + \frac{h}{1!} y'_n + \frac{h^2}{2!} y''_n + \dots$$

18. What is the error of Euler's method? **(Remembering)**

$$\text{Ans: Error at } (x=x_1) = \frac{h^2}{2!} y''(x_1, y_1), \text{ Error} = O(h^2) = \text{Error is the order of } h^2.$$

### PART-B

1. Find  $\frac{dy}{dx} = x^2 - y$ ;  $y(0) = 1$ , by Taylor series method to find the values of  $y(0.1)$ ,  $y(0.2)$ ,  $y(0.3)$  and  $y(0.4)$  assuming  $h = 0.1$ . **(Evaluating)**

2. Solve numerically  $\frac{dy}{dx} = x + y$  when  $y(1) = 0$  using Taylor's series upto  $x = 1.2$ ,  $h = 0.1$

**(Evaluating)**

3. Using Taylor series method to find  $y(0.1)$  if  $y' = x^2 + y^2$ ,  $y(0) = 1$ . **(Applying)**

4. Solve  $y' = \log(x + y)$ ,  $y(0) = 2$  by Modified Euler method,

Find the value of  $y(0.2)$ . **(Evaluating)**

5. Solve  $y' = xy + y^2$ ,  $y(0) = 1$ . Find (i)  $y(0.1)$  by Taylor's method (ii)  $y(0.2)$  by RK method (iii)  $y(0.3)$  by Euler's method (iv)  $y(0.4)$  by Milne's method. **(Evaluating)**

6. Using Modified Euler method find  $y$  when  $x = 0.1$  given that  $y(0) = 1$  and  $\frac{dy}{dx} = x^2 + y$ .

**(Applying)**

7. Apply the modified Euler's method to find  $y(0.2)$  and  $y(0.4)$ , given that  $y' = x^2 + y^2$ ,  $y(0) = 1$ . Take  $h = 0.2$ . **(Applying)**

8. Using RK fourth order solve  $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$  at  $x = 0.2$  given that  $y(0) = 1$ . **(Applying)**

9. Using Runge-kutta method of order 4, find  $y$  for  $x = 0.1, 0.2, 0.3$  given that

$$\frac{dy}{dx} = xy + y^2, y(0) = 1, \text{ and also find the solution at } x = 0.4 \text{ using Milne's method.}$$

**(Applying)**

10. Solve  $y' = x + y$ ;  $y(0) = 0$  by Runge-Kutta method of the fourth order to find  $y(0.2)$  and  $y(0.4)$  given that  $h = 0.2$ . **(Evaluating)**

11. Using Milne's method, find  $y(4.4)$  given  $5xy' + y^2 - 2 = 0$ . Given  $y(4) = 1$ ,  $y(4.1) = 1.0049$ ,  $y(4.2) = 1.0097$  and  $y(4.3) = 1.0143$ . **(Applying)**

## UNIT V-BOUNDARY VALUE PROBLEMS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

### PART-A

1. Write a note on the stability and convergence of the solution of the difference equation corresponding to the hyperbolic equation  $u_{tt} = a^2 u_{xx}$ . **(Remembering)**

Ans: For  $\lambda = 1/a$ , the solution of the difference equation is stable and coincides with the solution of the differential equation. For  $\lambda < 1/a$ , the solution is stable but not

convergent.

2. Fill up the blanks: **(Understanding)**

a) Explicit method is stable only if  $\lambda$  ..... ( $\lambda < 1/2$ )

b) Implicit method is convergent when  $\lambda$  ..... ( $\lambda = 1/2$ )

3. State the conditions for the equation;  $Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G$  where A, B, C, D, E, F, G are function of x and y to be (i) elliptic (ii) parabolic (iii) hyperbolic. **(Analyzing)**

Ans: The given equation is said to be

(i) Elliptic at a point (x,y) in the plane if  $B^2 - 4AC < 0$

(ii) Parabolic if  $B^2 - 4AC = 0$

(iii) Hyperbolic if  $B^2 - 4AC > 0$ .

4. What is the classification of  $f_x - f_{yy} = 0$ ? **(Evaluating)**

Ans: Here  $A = 0$ ,  $B = 0$ ,  $C = -1 \Rightarrow B^2 - 4AC = 0$  So the equation is Parabolic.

5. For what value of  $\lambda$ , the explicit method of solving the hyperbolic equation  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial u}{\partial t^2}$

is stable, where  $\lambda = \frac{c\Delta t}{\Delta x}$ ? Ans: when  $\frac{c\Delta t}{\Delta x} > 1$ . **(Evaluating)**

6. What is the error for solving laplace and poisson's equations by finite difference method? **(Remembering)**

Ans: The error in replacing  $\frac{\partial^2 u}{\partial x^2}$  by the difference expression is of the order  $O(h^2)$ . Since

$h = k$ , the error in replacing  $\frac{\partial^2 u}{\partial y^2}$  by the difference expression is of the order  $O(h^2)$ .

Hence the error for solving Laplace and Poisson equation is  $O(h^2)$ .

7. Write down the Crank – Nicholson formula to solve  $u_t = u_{xx}$ . **(Remembering)**

Ans: The Crank – Nicholson formula is  $u_{i+1,j+1} - 4u_{i,j+1} + u_{i-1,j+1} = u_{i+1,j} - u_{i-1,j}$ .

8. Write the diagonal five – point formula to solve the Laplace equation  $u_{xx} + u_{yy} = 0$ . **(Remembering)**

Ans: The diagonal five – point formula is  $u_{i,j} = \frac{1}{4} [u_{i,j} + u_{i+i,j} + u_{i,j-1} + u_{i-1,j}]$ .

## PART-B

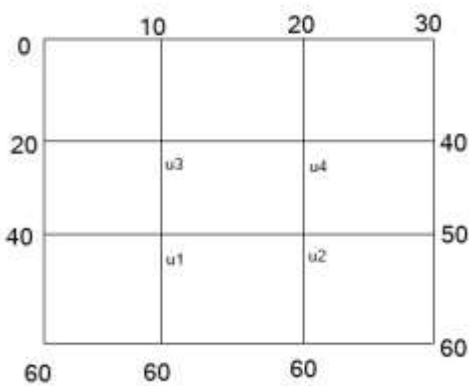
1. Solve by Crank-Nicholson method the equation  $u_{xx} = u_t$  subject to  $u(x, 0) = 0$ ,

$u(0,t) = 0$  and  $u(1,t) = t$ , for two time steps By Crank-Nicholson method solve the

equation  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$  subject to  $u(x, 0) = 0$ ,  $u(0, t) = 0$  and  $u(1, t) = t$  for two time steps.

**(Evaluating)**

2. solve  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ ,  $0 < x < 2$ ,  $t > 0$  given  $u(0, t) = u(2, t) = 0$ , and  $u(x, 0) = \sin \frac{\pi x}{2}$ ,  $0 \leq x \leq 2$ ,  $\Delta x = 0.5$  and  $\Delta t = 0.25$  using for two time steps by Crank-Nicholson implicit difference method. **(Evaluating)**
3. Approximate the solution to the wave equation  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ ,  $0 < x < 1$ ,  $t > 0$ ,  $u(0, t) = u(1, t) = 0$ ,  $t > 0$ ,  $u(x, 0) = \sin 2\pi x$ ,  $0 \leq x \leq 1$  and  $\frac{\partial u}{\partial t}(x, 0) = 0$ ,  $0 \leq x \leq 1$  with  $\Delta x = 0.25$  and  $\Delta t = 0.25$  for 3 time steps. **(Evaluating)**
4. Using Crank-Nicholson's scheme, solve  $u_{xx} = u_t$ , given  $u(x, 0) = 0$ ,  $u(0, t) = 0$ ,  $u(1, t) = t$ , for two time steps, taking  $h = 1/4$  in x-direction and  $k = 1/16$  in t-direction. **(Applying)**
5. Evaluate  $y_{tt} = y_{xx}$  upto  $t = 0.5$  with a spacing of 0.1 subject to  $y(0, t) = 0$ ,  $y(1, t) = 0$ ,  $y_t(x, 0) = 0$  and  $y(x, 0) = 10 + x(1 - x)$ . **(Evaluating)**
5. Solve the Laplace equation  $u_{xx} + u_{yy} = 0$  over the square mesh of side 4 units satisfying the following boundary conditions:
- (i).  $u(0, y) = 0$  for  $0 \leq y \leq 4$
  - (ii).  $u(4, y) = 12 + y$  for  $0 \leq y \leq 4$
  - (iii).  $u(x, 0) = 3x$  for  $0 \leq x \leq 4$
  - (iv).  $u(x, 4) = x^2$  for  $0 \leq x \leq 4$  (Two iterations are enough) **(Evaluating)**
6. Solve the Poisson equation  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh with sides  $x = 0, y = 0, x = 3$  and  $y = 3$  with  $u = 0$  on the boundary and mesh length 1 unit. **(Evaluating)**
7. Solve the Poisson equation  $\nabla^2 u = 8x^2y^2$  over the square mesh with sides  $x = -2, x = 2, y = -2, y = 2$  with  $u = 0$  on the boundary and mesh length 1 unit. **(Evaluating)**
8. Solve  $\nabla^2 u = 0$ , the boundary conditions are given below: **(Evaluating)**



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