K.S.R. COLLEGE OF ENGINEERING (Autonomous)

R 2018

SEMESTER - II

MA18232

OPERATIONS RESEARCH FOR MANAGEMENT (Master of Business Administration)

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Objective(s):

- To learn about the optimization techniques for decision making problems in management studies.
- To understand concepts of Transportation and Assignment problems and also to study the concepts of project scheduling by network analysis.
- To enumerate the concepts in stock control models and to understand the concepts of scheduling and Replacement problems in business.

UNIT - I LINEAR PROGRAMMING PROBLEMS

[12]

Introduction to applications of operations research in functional areas of management. Definition - Linear Programming Problem - Formulation of Linear Programming Problems - graphical and simplex method using slack variables.

TRANSPORTATION AND ASSIGNMENT PROBLEMS

[12]

Definition - Transportation Models (Minimizing and Maximizing Cases) Definition - Balanced and Unbalanced cases -Definition - Initial Basic feasible solution by North West Corner Rule, Least cost and Vogel's approximation methods. Optimality by Modified method. Definition and basic concepts of Assignment Models (Minimizing and Maximizing Cases) - Balanced and Unbalanced Cases. Travelling Salesman Problem.

CPM / PERT ANALYSIS

Definition - Critical path method - Definition - Project Evaluation and Review Techniques (PERT) analysis - Cost consideration in PERT / CPM and problems.

INVENTORY MODELS AND GAME THEORY

[12]

Definition and examples of Inventory Models - EOQ and EBQ Models (With and without shortages), Quantity Discount Models (one price break and two price breaks) and problems.

Definition and concepts in Game Theory – Two person zero sum games with Saddle point – without Saddle point method using Dominance rule – problems - Algebraic method, Graphical and linear programming solutions.

SEQUENCING PROBLEMS AND REPLACEMENT MODELS

Basic definitions and concepts in Sequencing problems - n Jobs with two machines - n Jobs with m Machines - Two Job with m Machines and problems. Definitions and examples of Replacement Models - Individual replacement Models (With and without time value of money) - Group Replacement problems.

Total (L:45 T:15) = 60 Periods

Course Outcomes: On Completion of this course, the student will be able to

- To understand the scope and role of OR in business administration and formulation of a managerial decision problem into a mathematical model and apply various methods to optimize the linear programming problem.
- To analyze the minimum transportation cost and assignment cost to produce maximum profit in business.
- To construct an effective project design by CPM and apply the concept of PERT for a real time project to analyze completion time.
- To ensure the control over the production management, inventory control systems is applied.
- To understand the concept of sequencing of machines to optimize the total elapsed time, idle time of the machines by applying in business.

Reference Books:

- 1. P K Gupta & Man Mohan Singh, Problems in Operations Research, Sultan Chand 12th edition, 2014
- Hamdy A Taha, Introduction to Operations Research, Prentice Hall India, Seventh Edition, 9th edition 2013.
- 3. Hira and Gupta "Problems in Operations Research", S.Chand and Co, 2012.
- N.D. Vohra "Quantitative Techniques in Management" 4th edition Tata McGraw Hill Education(p)ltd.

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K.S.R. COLLEGE OF ENGINEERING (AUTONOMOUS), TIRUCHENGODE – 637 215 DEPARTMENT OF BUSINESS ADMINISTRATION (MBA)

NAME : Ms. DHAVAMANI

CLASS : I MBA

SUBJECT : MA18232 - OPERATIONS RESEARCH FOR MANAGEMENT

UNIT -I: LINEAR PRPGRAMMING PROBLEM

2 Mark Questions:

1 Define the term operation research.

Operations Research is the application of scientific methods, techniques and tools to operations of systems to obtain optimal solution to the problems; it provides a quantitative technique to the managers for making better decisions for operations under control.

2 Give any tow applications of operation research in business/management.

Production, blending, product mix.

Inventory control, demand forecast, sale and purchase.

Transportation, repair and maintenance, scheduling and sequencing.

Planning, scheduling and controlling of projects.

Optimal allocation of men, machines, materials, time and money.

Location and size of warehouses, distributions centers, retail depots, etc.

3 When a linear programming problem is said to be unbounded?

If the values of the objective function Z can be increased or decreased indefinitely, such solutions are called unbounded solutions in LPP>

4 Profit for product A, B and C is Rs.30, Rs.45 and Rs. 90. Respectively. Write an objective function.

Le x_1 , x_2 and x_3 be the profit from the product A, B and C, then the objective function is Max $Z = 30x_1 + 45x_2 + 90x_3$

5 What are the components of LPP?

The components of LPP

- (i) Decision variable
- (ii) Objective function
- (iii) Constraints
- (iv) Non-negative restrictions

6 What is meant LPP?

Linear programming problem deals with the optimization (maximization or minimization) of a function of decision variables subject to a set of constraints.

7 Mention the use of artificial variable in solving LPP.

Any non negative variable which is introduced in the constraint in order to get the initial basic feasible solution is called artificial variable.

8 What is feasible region?

A region in which all the constraints are satisfied simultaneously is called a feasible region.

9 Why slack/surplus variables are used in LPP?

The non-negative variable which is added to LHS of the constraint to convert the inequality \leq into an equation is called slack variable. i.e. $\sum_{j=1}^{n} a_{ij} x_i + s_i = b_i (i = 1, 2, ..., m)$ where s_i are called slack variables.

The non-negative variable which is subtracted from the LHS of the constraint to convert the inequality \geq into an equation is called surplus variable. i.e. $\sum_{j=1}^{n} a_{ij} x_i - s_i = b_i (i = 1, 2, ..., m)$ where s_i are called surplus variables.

10 Define 'objective function" and 'Constraint".

It is a function of decision variables whose value must be optimized (maximized or minimized) It is a set of simultaneous linear equations (or in equalities).

11 **Define principle of duality**.

For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem

12 The advertising budget is Rs. 50,000/- advertising in magazine and in newspaper costs Rs. 4500/- and 6000/- respectively. Formulate a constraint equation.

Le x_1 , x_2 denotes the advertising in magazine and news paper respectively then the constrains are $4500x_1 + 6000x_2 \le 500000$

13 Write the canonical form of LPP?

The general form of LPP can be expressed in the canonical form as follows,

n

maximize $Z=\Sigma$ cj xj, subject to Σ aij xj<=bi (i=1,2,3,....m) and xj>=0. i=1

14 Write a note on problem formulation.

The formulation of an LPP consist the following steps

- Identify the decision variable
- Formulate the objective function
- Write down the constraints
- Mention the non-negative restrictions.

15 What is the difference between a feasible solution and optimal solution of LPP?

A solution to the LPP which satisfies the non-negativity restrictions of the LPP is called a feasible solution. Any feasible solution which optimizes the objective function is called its optimal solution.

16 What is degeneracy?

The concept of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy. In the case of a BFS, all the non basic variables have zero value. If some basic variables also have zero value, then the BFS is said to be a degenerate BFS.

17 What are the methods used to solve an LPP involving artificial variables?

- Big M method or penalty cost method
- Two-phase simplex method

18 **Define dual of LPP.**

For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem. If the primal problem is

Maximize Z = CXsubject to $AX \le b$ $X \ge 0$

Then the dual is

Minimize
$$Z^* = b^T Y$$

subject to $A^T Y \ge C^T$
 $Y \ge 0$

19 Write the general mathematical model of LPP in matrix form.

Max or Min Z = CXSubject to AX ($\leq = \geq$) b $X \ge 0$

12 Mark Questions:

- A company has three operational departments (weaving, processing and packing) with capacity to produce three different types of clothes namely suiting's, shirting and woollens yielding a profit of Rs. 2, Rs. 4 and Rs. 3 per metre respectively. One metre of suiting requires 3 minutes in weaving, 2 minutes in processing and 1 minute in packing. Similarly one metre of shirting requires 4 minutes in weaving, 1 minute in processing and 3 minutes in packing. One metre of woollen requires 3 minutes in each department. In a week, total run time of each department is 60, 40 and 80 hours for weaving, processing and packing respectively. Formulate the linear programming problem to find the product mix to maximize the profit.
- Case study: A garment manufacturing company produces two types of T-shirts, with collar and without collar. The shirts must undergo cutting and stitching processes. Shirts with collar requires, 159 minutes cutting and 25 minutes of stitching. Shirts without collar requires, 10 minutes of cutting and 20 minutes of stitching. The full time available for cutting is 480 minutes, but only 360 minutes is available for stitching. Profit for each shirt with and without collar is Rs. 30/- and Rs.20/- respectively. Determine how many shirts with collar and without collar should be manufactured to maximize the profit?
- Case study: XYZ apparels manufacture branded jeans. Three jeans A, B and C are manufactured. The manufacture of brand A requires 3 minutes of machine time, 30 minutes of labour and costs Rs. 700. Brand B requires 3.5 minutes machine time, 40 minutes labour and costs Rs. 1000/- to produce. Brand C requires 4 minutes machine time and 1 hour labour and costs Rs. 1300/- to produce. Brand A sells for Rs. 1200, brand B for Rs 1400 and brand C for Rs. 2000. The company 20 works on a 5 days a week schedule with two shifts of 8 hours each. It has four machines available for production and 50 employees on each shift. Its weekly manufacturing budget is Rs. 10 lakhs. It is objective is to maximize its profit using suitable product mix of three brands Formulate the problem as LPP and solve the problem using graphical method.
- Case study: The Bitz-Karan Corporation faces blending decision in developing a new cat food called Yum-Mix. Two basic ingredients have been combined and tested, and the firm has determined that to each can Yum-Mix al least 30 units of protein and at least 880 units of riboflavin must be added. These two nutrients are available in two compositing brands of animal food

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supplements. The cost per kilogram of brand A is Rs.9, and the cost per kilogram of brand B supplement is Rs.15. A kilogram of brand A added to each production batch of Yum-Mix provides a supplement of 1 unit of protein and 1 unit of riboflavin to each can. A kilogram of brand B provides 2 units of protein and 4 units of riboflavin in each can. Bitz- Karan must satisfy these minimum nutrient standards while keeping costs of supplements to a minimum.

- a) Formulate this problem to find the best combination of the two supplements to meet the minimum requirement at the least cost.
- b) Find the optimal solution by simplex method.
- Case study: A company makes two kinds of leather belts. Belts A is high quality belt, and belt B is of lower quality. The respective profits are Rs 4 and Rs. 3 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 per day are available. There are only 700 buckles available for belt B. Determine the optimal product mix using graphical method.
- 6 Case study: Let us assume that you have inherited Rs. 1, 00, 000 from your father-in-law that can be invested in a combination of only two stock portfolios, with the maximum investment allowed in either portfolio set at Rs. 75,00. The first portfolio has an average rate of return of 10% whereas the second has 20%. In terms of risk factors associated with these portfolios, the first has first has a risk rating of 4 (on a scale from 0 to 10), and the second has 9. Since you wish to maximize your return, you will not accept on average rate of below 12% or a risk factor above 6. Hence, you then face the important question. How much should you invest in each portfolio? Formulate this as a linear programming problem and solve it by graphic method.
- 7 Use graphical method to solve the following LPP:

Minimize $Z = -x_1 + 2x_2$

Subject to the constraints:
$$-x_1 + 3x_2 \le 10$$

 $x_1+x_2 \leq 6$

$$x_1 - x_2 \le 2$$
 and $x_1, x_2 \ge 0$

8 Use graphical method to solve the following LPP:

Maximize $Z = 2x_1 + 3x_2$

Subject to the constraints:
$$x_1 + x_2 \le 30$$

 $x_2 \ge 3$

$$x_1 - x_2 \ge 0$$

$$0 \le x_1 \le 20$$
 and $0 \le x_2 \le 12$

9 Use graphical method to solve the following LPP: (Special Case: Alternate optimum Solution)

Maximize
$$Z = 2x_1 + 4x_2$$

Subject to the constraints: $x_1 + 2x_2 \le 5$

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$$x_1 + x_2 \le 4$$
 and $x_1, x_2 \ge 0$

10 Use graphical method to solve the following LPP: (Special case: Unbounded solution)

Maximize $Z = 6x_1 + x_2$

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Subject to the constraints: $2x_1 + x_2 \ge 3$

$$X_{2-} x_1 \ge 0$$
 and $x_1, x_2 \ge 0$

11 Use graphical method to solve the following LPP: (special case: infeasible solution)

Maximize $Z = x_1 + x_2$

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Subject to the constraints: $x_1 + x_2 \le 1$

$$-3x_{1} + x_{2} \ge 3$$
 and $x_{1}, x_{2} \ge 0$

12 Solve the Following LPP by simplex method

(a) Maximize $Z = 4x_1 + 10x_2$

Subject to
$$2x_1 + x_2 \le 10$$

$$2x_1 + 5x_2 \le 20$$

and $x_1, x_2 \ge 0$

(b) Maximize Z = 3x + 2y + 5z

Subject to
$$x + 4y \le 420$$

$$3x + 2z \le 460$$

$$x + 2y + z \le 430$$
 and $x, y, z \ge 0$

(c) Minimize Z = x - 3y + 2z

Subject to
$$3x - y + 2z \le 7$$

$$-4x + 3y + 8z \le 10$$

$$-2x + 4y \le 12$$
 and x, y, $z \ge 0$

(d) Maximize $Z = 107x_1 + x_2 + 2x_3$

Subject to:
$$14x_1 + x_2 - 6x_3 + 3x_4 = 7$$

$$16x_1 + x_2 - 6x_3 \le 5$$

$$3x_1-x_2-x_3\leq 0$$

and
$$x_1, x_2, x_3, x_4 \ge 0$$

13 Solve the following LPP by Big M or penalty method

(a)
$$Max Z = x_1 + 2x_2 + 3x_3 -$$

(b) Maximize
$$Z = 6x_1 + 4$$

(b) Maximize
$$Z = 6x_1 + 4x_2$$
 (c) Maximize $Z = 3x_1 + 2x_2$

$$S.T: x_1 + 2x_2 + 3x_3 = 15$$

Subject to
$$2x_1 + 3x_2 \le 30$$
 Subject to $2x_1 + x_2 \le 2$
 $3x_1 + 4x_2 \ge 12$

$$2x_1 + x_2 + 5x_3 = 20$$

$$3x_1 + 2x_2 \le 24$$

and
$$x_1, x_2 \ge 0$$

$$x_1 + x_3 + x_4 = 10$$
 and $x_1, x_2, x_3, x_4 \ge 0$

$$x_1 + x_2 \ge 3$$
 and $x_1, x_2 \ge 0$

14 Solve the Following LPP by Two-phase method

(a) Maximize
$$Z = -4x-3y-9z$$

subject to
$$2x+4y+6z \ge 15$$

 $6x+y+6z \ge 12$

and
$$x, y, z \ge 0$$

(b) Maximize
$$Z = 5x + 8y$$

Subject to
$$3x + 2y \ge 3$$

$$x+4y\!\ge\!4$$

$$x + y \le 5$$

and x,
$$y \ge 0$$

UNIT - II TRANSPORTATION AND ASSIGNMENT PROBLEMS

2 Mark Questions:

Define feasible and basic solution in transportation problem.

Any set of non negative allocations x_{ij} which satisfies the row and column sum requirements is called a feasible solution. A feasible solution is called a basic feasible solution if the number of non negative allocations is equal to m+n-1

What do you mean by degeneracy in transportation?

If the number of non negative independent allocations in a m×n T.P is less than m+n-1, then there exists degeneracy in the solution of the T.P.

What is the difference between TP and Assignment problem?

The transportation problem deals with the transportation of a product manufactured at different plants or factories(supply origins) to a number of different warehouses(demand destinations). The objective is to

satisfy the destination requirement within the plant capacity at minimum transportation cost.

Given n facilities and n jobs and given the effectiveness of each facility for each job, the problem is to assign each facility to one only job so as to optimize the given measure of effectiveness.

Examples: products to factories

Jobs to Machines

4 Find the initial transportation cost using least cost method

	a	b	c	supply
I	7	4	4	8
II	4	0	5	12
III	6	5	3	10
	Demand	10	14	6

5 Differentiate between balance and unbalanced cases in assignment models.

If the number of rows is not equal to the number of columns in the cost matrix of the assignment problem, then the given assignment problem is said to be unbalanced.

If the number of rows is equal to the number of columns in the cost matrix of the assignment problem, then the given assignment problem is said to be balanced.

6 How maximization problem can be handled in transportation problems?

The maximization problem in transportation problem can be converted into a minimization problem and then applied the MODI method to solve the problem

7 A manager has to ship products from 3 factories to 4 warehouses. The supply from factories are 125, 75, 200 units and demand for warehouses are 100, 175, and 125. State whether the transportation model is a balanced one or not.

8 How does the problem of degeneracy arise in a transportation problem?

If the number of non negative independent allocations in a $m \times n$ T.P is less than m+n-1, then there exists degeneracy in the solution of the T.P. This may be arising in getting initial solution or at any stage of optimality by MODI method.

9 What do you understand by unbalanced transportation problem?

Any T.P is said to be unbalanced if
$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$
. i.e., if the total supply is not equal to the total

demand.

10 How the problem of prohibited route in a transportation problem is solved?

The prohibited route problem can be solving by MODI method as usual except that the prohibited route could not have any allocations.

11 What are the objectives of travelling salesmen problem?

The objective of the travelling salesman problem is that the salesman has to visit various cities, each city exactly once, and return to the starting place in such a way that his total travelling time is minimized.

12 Solve the following assignment problem

	Men					
		1	2	3		
Job	A	8	0	6		
	В	5	6	-		
	С	4	0	7		

exist.

14 State whether the travelling salesmen problem has a solution

City	a	b	c	d
a			X	
b				X
c		X		
d	X			

12 Mark Questions:

1 Solve the following TP by finding the starting basic feasible solution by VAM

Origins	Destinations					
Origins	D1	D2	D3	DA	Capacity	
O1	21	16	25	13	11	
O2	17	18	14	23	13	
O3	32	27	18	41	19	
Demand	6	10	12	15		

A military equipment is to be transported from origins x, y, z to destinations A, B, C and D. the supply at the origins, the demand at the destinations and the time of shipment is shown in the table. Work out a transportation plan so that the time required for shipment is the minimum

Origins	Destinations					
Origins	A	В	С	D	Supply	
X	10	22	0	22	8	
у	15	20	12	8	13	
Z	21	12	10	15	11	
Demand	5	11	8	8		

- Case Study: The Hardrock concrete company has plants in three locations and is currently working on three major construction projects each located at a different site. The shipping cost per truckload of concrete, daily plant capacities, and the daily project requirements are provided in the table below
 - (a) Formulate an initial feasible solution to Hardrock's transportation problem using North West corner rule. Then evaluate each unused shipping route by computing all improvement indices. Is this solution optimal? Why?
 - (b) What is the optimal solution and associated cost?

To From	Project A	Project B	Project C	Project Capacities
Plant1	10	4	11	70
Plant2	12	5	8	50
Palnt3	9	7	6	30
requirement	40	50	60	150

4 Special case1: unbalanced TP Problem

A product is produced by four factories A, B C and D. Their production capacities are: factory A-50 units, B-70 units C-30 units and D-50 units. These factories supply the product to four stores, demand of which is 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table below.

Factory	Stores					
ractory	s1	s2	s3	s4		
F1	2	4	6	11		
F2	10	8	7	5		
F3	13	3	9	12		
F4	4	6	8	3		

5 **Special case1: unbalanced TP Problem:** A company has received a contract to supply gravel to 12 three new construction projects located in towns A, B and C. construction engineers have estimated the required amounts of gravel which will needed at these construction projects as follows:

Project location	A	В	C
Weekly	72	102	41
requirement			
(No. of truck			
loads)			

The company has three gravel pits located in towns X, Y and Z. the amount of gravel that can be supplied from these pits per week is

Pit	X	Y	Z
Truck	76	82	77
loads			

The construction costs from pits to construction sites per truck load are

	TO				
		A	В	С	
From	X	4	8	8	
	Y	16	24	16	
	Z	8	16	24	

Prepare a distribution of gravel schedule from pits to project sites, so that the transportation cost is minimum. Also find the total transportation cost.

6 **Special case1: unbalanced TP Problem**: Find the initial transportation cost using Vogel's 12 approximation method and obtain the allocation from factories to destination points

Origins	Destinations						
Origins	D1	D2	D3	D4	D5	Capacity	
F1	4	13	12	5	9	7	
F2	3	6	9	4	3	11	
F3	5	11	6	10	14	12	
Demand	4	6	6	7	9		

7 Special case2: Maximization type problem:

ABC Enterprises is having three plants manufacturing dry-cells, located at different locations. Production cost differs from plant to plant. There are five sales offices of the company located in

different regions of the country. The sales prices can differ from region to region. The shipping cost from each plant to each sales office and other data re given below:

Dlont	Product	Production data			shipping costs (In Rs)				
Plant Production cost/unit (Ra)	Max. Capacity No. of units	A	В	С	D	Е			
Plant1	20	150	1	1	5	9	4		
Plant2	22	200	9	7	8	3	6		
Plant3	18	125	4	5	3	2	7		

Demand and sales prices

Demand	80	100	75	42	125
Sales prices	30	32	31	34	29
(Rs)					

Find the production and distribution schedule most profitable to the company.

8 **Special case2: Maximization type problem:** Maximize the following TP using highest profit 12 method:

	D1	D2	D3	D4	D5	D6	Supply
F1	35	41	28	16	20	12	32
F2	14	21	28	30	15	24	180
F3	45	18	17	29	26	19	200
Demand	125	125	100	100	175	75	

9 **Special case3: Alternate optimum solution:** A company manufacturing air-coolers has two plants located at Mumbai and Kolkata with a weekly capacity of 200 units and 100 units, respectively. The com-any supplies air-coolers to its 4 showrooms situated at Ranchi, Delhi. Lucknow and Kanpur which have a demand of 75, 100, 100 and 30 units respectively. The cost of transportation per unit (in Rs) is shown in the following table:

	Ranchi	Delhi	Lucknow	Kanpur
Mumbai	90	90	100	100
Kolkata	50	70	130	85

Plan the production programme so as to minimize the total cost of transportation.

10 In a job shop operation, four jobs may be performed on any of four machines. The hours required for each job on each machine are presented in the following table. The plant supervisor would like to assign jobs so that total time is minimized. Use the assignment method to find the best solution.

Job	Machine				
	W X Y Z				
A12	10	14	16	13	
A15	12	13	15	12	
B2	9	12	12	11	
B9	14	16	18	16	

			M	en		
		1	2	3	4	5
	A	18	14	X	13	17
Job	В	15	17	13	18	20
	С	22	23	20	21	19
	D	18	X	15	18	14
	Е	22	21	20	19	21

12 **Special case2: Maximization type problem:** A company has a team of four salesmen and four districts where the company wants to starts its business. After taking into account the capabilities of sale men and the nature of districts, the company estimates that the profit per day in hundreds of rupees for each salesman in each districts as below. Find the assignment of salesmen to various district which will yield maximum profit

		Districts					
		1	2	3	4		
salesman	A	16	10	14	11		
	В	14	11	15	15		
	C	15	15	13	12		
	D	13	12	14	15		

13 **Special case2: Travelling salesmen problem:** A salesman must travel from city to city to 12 maintain his accounts. This week he has to leave his home base and visit other cities and return home. The table shows the distances (in Kms) between the various cities. The home base is city A.

		To city					
		A	В	С	D	Е	
From	A		40	70	90	30	
city	В	80		50	100	40	
	С	100	90		60	90	
	D	140	60	80		70	
	Е	30	50	40	200		

Determine the tour that will minimize the total distance of visiting all cities and returning home.

A salesman must travel from city to city to maintain his account. This week he has to leave his home base and visit each other city and return home. The table shows the distances (in Km) between the various cities. The home city is city A. use the assignment method to determine the tour that will minimize the total distance of visiting all cities and returning home

From	To Cities				
Cities	A	В	С	D	Е
Α	-	375	600	150	190
В	375	-	300	350	175
С	600	300	-	350	500
D	160	350	350	1	500
Е	190	175	500	300	-

15 **Special case2: Maximization type problem:** Solve the following assignment problem for maximization:

		Machines				
		P	Q	R	S	
jobs	A	51	53	54	50	
Jous	В	47	50	48	50	
	С	49	50	60	61	
	D	63	64	60	60	

UNIT - III CPM / PERT ANALYSIS

2 Mark Questions:

	1	Write any	rules f	or construct	ing network?
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Each activity is represented by one and only one arrow.

Time flows from left to right.

Arrows should be straight and not curved.

2 Define Critical path method and mention its advantages.

The critical path of a network gives the shortest time in which the whole project can be completed. It is the chain of activities with the longest time duration. Any delay in any of the activities results in the delay of the completion of the project.

3 Differentiate between CPM and PERT.

	PERT	CPM
1	Event oriented	Activity oriented
2	Probabilistic	Deterministic
3	Three time estimate namely optimistic, pessimistic and most likely are given	Time fixed
4	Resources such as labour, equipments, materials are limited	No limitations of resources

I J
F,G
pessimistic time
14
lkerson's rule?

Write the procedure for numbering the events in a network diagram using Fulkerson's rule? Step1; Number the start or initial event as 1.

Step2: From event 1, strike off all outgoing activities. This would have made one or more events as initial events (event which do not have incoming activities). Number that event as 2.

Step3: Repeat step 2 for event 2, event 3 and till the end event. The end event must have the highest number

- 7 What are the errors in network construction?
- 8 Find the expected duration and standard deviations of the following activity

Activity	Optimistic time	most likely time	pessimistic time
A	1	1	7

- 9 The crash and normal cost is Rs. 580 and Rs. 430 for an activity whose crash time and normal time is 8 days and 10 days respectively. Calculate the cost slope.
- 10 Draw a network diagram for the activities of a project

Activity: A B C D E Immediate Predecessor: -- A A C

- 11 What are the three time estimates associated with PERT?
 - (i) The optimistic time estimates.
 - (ii) The pessimistic time estimates and
 - (iii) The most likely time estimates.
- 12 The optimistic, pessimistic and most likely time estimates for an activity is 12 days, 18 days and 14 days respectively. Calculate the average time of the activity.

13 **Define a network.**

A Network is a symbolic representation of the essential characteristics of a project. Network technique is a tool of project management. PERT and CPM are the widely applied techniques.

14 Define the term: Total, Free and Independent float and dummy activity.

Total Float: The total float of an activity is the difference between the latest start time and the earliest start time of that activity. TFij = LS $_{ij}$ - ES $_{ij}$ or TF $_{ij}$ = (TL - TE) - t_{ij}

Free Float (FFij): The time by which the completion of an activity can be delayed from its earliest finish time without affecting the earliest start time of the succeeding activity is called free float.

$$FF_{ij} = (Ej - Ei) - t_{ij}$$
 (or) $FF_{ij} = Total$ float – Head event slack

Independent Float (IFij): The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time. IF ij = (Ej - Li) - tij (or) IFij = Free float – Tail event slack **Dummy activity**: Certain activity, which neither consumes time nor resources but are used simply to present a connection between events are known as dummy activities. When two activities have the same head and tail events, this cannot be represented in a network diagram without using dummy activity.

- 15 If A precedes B, C and D succeeds B, E has no predecessor and C precedes F. construct a network diagram for the activities.
- Define the following terms: a) Activity b) Event C) Predecessor and Successor activity d) Float or slack

Activity: An activity represents an action and consumption of resources (time, money, energy) required to complete a portion of a project. Activity is represented by an arrow.

Event: An event (or node) will always occur at the beginning and end of an activity. The event has no resources and is represented by a circle. The ith event and jth event are the tail event and head event respectively.

17 What are the two basic planning and control techniques in a network analysis.

Critical Path Method (CPM)

Programme Evaluation and Review Technique (PERT)

18 When do you call an activity critical?

An activity is said to be critical if a delay in its start will cause a further delay in the completion of the entire project.

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12 Mark Questions:

1 Consider the data of the project, find its critical path and project duration with the following relations:

A < D; A < E; B < F; D < F; C < G; C < H; F < I; G < I

Activity	A	В	С	D	Е	F	G	Н	I
Duration(days)	8	10	8	10	16	17	18	14	9

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2 The following table shows the jobs of a project with their time duration in days. Draw the network and determine the critical path. Also calculate all the floats.

Jobs	1-	1-	1-	2-	3-	4-	5-	5-	6-	6-	7-	8-	9-	10-	11-
	2	3	4	5	7	6	7	8	7	9	10	10	10	11	12
Duration	10	8	9	8	16	7	7	7	8	5	12	10	15	8	5

Consider the data of the project, find its critical path and project duration:

Activity	A	В	С	D	Е	F	G	Н	I
Predecessor	-	-	A	В	C,D	В	Е	Е	F,G
Duration(days)	4	7	2	9	6	5	2	10	4

4 A small project consists of 11 activities A, B, C.....K. The precedence relationship is given as: A, B can start simultaneously, A < C, D, I: B < G, F: D < G, F: F < H, K: G, H < J: I, J, K < E. The duration of the activities is as follows. Draw a network of the project. Summarize the CPM calculation and find Critical path.

Activity	A	В	C	D	Е	F	G	Н	I	J	K
Duration(days)	5	3	10	2	8	4	5	6	12	8	9

5 A project consists of eight activities with the following time estimates.

Activity	A	В	C	D	Е	F	G
predecessor	-	A	A	В,С	В	D,E	D
Optimistic time	1	2	3	4	3	2	4
Pessimistic time	7	14	3	22	15	14	4
Most likely time	3	6	3	10	7	5	4

- (i) Draw PERT Network
- (ii) Find the expected time for each activity and determine the critical path
- (iii) What project duration will have 95% confidence of completion?

6 A project consists of eight activities with the following time estimates.

Activity	A	В	С	D	Е	F	G
predecessor	-	-	-	Α	В	C	D,E
Optimistic time	1	1	2	1	2	2	3
Pessimistic time	1	4	2	1	5	5	6
Most likely time	7	7	8	1	14	8	15

- (i) Draw PERT Network
- (ii) Find the expected time for each activity and determine the critical path
- (iii) What is the probability that the project will be completed in 22 days

A project consists of eight activities with the following time estimates.

<u> </u>								
Activity	1-2	2-3	2-4	3-5	4-6	5-6	5-7	6-7
Optimistic time	3	3	2	4	4	0	3	2
Pessimistic time	3	9	6	8	8	0	5	8
Most likely time	3	6	4	6	6	0	4	5

- (i) Draw PERT Network
- (ii) Find the expected time for each activity and determine the critical path
- (iii) What is the probability that the project will be completed in 23 days

8 **Case Study:** Determine the optimum project duration and cost for the following data. Indirect cost is Rs.70 per day.

Activity	Norm	al	Crash		
Activity	Time	Cost	Time	Cost	
1-2	8	100	6	200	
1-3	4	150	2	350	
2-4	2	50	1	90	
2-5	10	100	5	400	
3-4	5	100	1	200	
4-5	3	80	1	100	

9 Case Study: The following table gives data on normal time-cost and crash time-cost for a project: 20

Act	ivity	1-2	1-3	2-4	2-5	3-4	4-6	5-6	6-7
Normal	Time (days)	6	4	5	3	6	8	4	3
Norman	Cost (Rs)	600	600	500	4510	900	800	400	450
Crash	Time (days)	4	2	3	1	4	4	2	2
Crasii	Cost (Rs)	100	2000	1500	650	2000	3000	1000	800

The indirect cost per day is Rs. 100.

- (i) Draw the network and identify the critical; path.
- (ii) What are the normal project duration and associated cost?
- (iii) Crash the relevant activities systematically and determine the optimum project completion time and cost.

UNIT -IV INVENTORY MODEL AND QUEUEING THEORY

2 Mark Questions:

- 1 What are the costs involved in inventory problems?
 - Set up cost:
 - Ordering Cost:
 - Set up cost:
 - Ordering Cost:
 - Purchase cost (or) Production cost:
 - Carrying (Holding) Cost:
 - Shortage (Stock out) Cost:
- 2 Define the terms: **Lead time** and **Reorder level.**

Lead Time: The time gap between placing of an order and its actual arrival in the inventory is known as lead time.

Reorder level: when the stock of items falls to a reorder level, which is equal to the lead time requirement plus safety stock.

- 3 What are the types of inventories?
 - Finished Goods
 - Work-in-Process
 - Raw Material

4 Define the terms: **Order cycle** and **Time Horizon.**

Order Cycle: An order cycle is identified by the time period between two successive order placements. **Time Horizon:** The time period over which size of inventory will be controlled is called the time horizon.

5 **Define saddle point.**

A saddle point is the position in the payoff matrix, where the maximum of row minima coincides with the minimum of column maxima.

6 Differentiate between pure and mixed strategy.

Pure Strategy: it is the decision rule which always used by the player to select the particular strategy to maximize gain or minimize losses.

Mixed Strategy: courses of action that are to be selected on a particular occasion with some fixed probability are called mixed staratgey.

7 What is meant by quantity discount models?

It is a common practice that consumers are encouraged for bulk purchasing by allowing some extra discounts is quantity discount models

8 Define the theory of games.

The competitive situation will be called a game, if it has the following properties:

- (i) There is a finite number of participants called players.
- (ii) Each player has a finite number of strategies available to him.
- (iii) Every game results in an outcome.

Give the assumptions for the manufacturing model without shortages.

- Demand is continuous and at a constant rate
- Set up cost is fixed
- Inventories are made at regular interval

10 What do you meant by two person zero-sum game?

A game with two players, where a gain of one player equals the loss of the other is known as a two-person zero sum game.

11 What are the various types of games?

- Two person game
- n- person game
- Zero sum game
- Non zero sum game

12 Define the terms: Maxmin and Minimax principles.

Maximin Principle: For a player A minimum value in each row represents the least gain to him if he chooses his particular strategy and he will then select the strategy that gives largest gain among the row minimum values is called maximin principle for player A.

Minimax Principle: For a player B maximum value in each column represents the maximum loss to him if he chooses his particular strategy and he will then select the strategy that gives minimum loss among the column maximum values is called minimax principle for player B.

13 State the assumptions underlying game theory.

- Each player has available to him a finite number of possible strategies
- Players acts rationally and intelligently
- One player attempts to maximize gains and other attempts to minimize losses.

14 Mention the reasons for carrying inventory.

• Improve customer service

- Reduce cost maintenance of operational capability
- Avoiding over stock

15 What are the objectives of inventory?

- It satisfies the demand of consumers.
- It identifies the optimum size of inventory that will serve to minimize the costs and maximize the return on investment.

16 State any two factors affecting inventory.

Following are the factors are affecting inventory:

- Relevant inventory costs
- Demand for inventory items
- Replenishment lead time
- Length of planning period
- 11 Define the terms: set up cost, ordering cost, carrying sot, shortage cost, salvage cost and revenue cost.

Set up cost: This is the cost associated with the setting up of machinery for production. It is independent of the quantity ordered or produced.

Ordering Cost: Costs associated with ordering of raw material for production process.

Set up cost: This is the cost associated with the setting up of machinery for production. It is independent of the quantity ordered or produced.

Ordering Cost: Costs associated with ordering of raw material for production process.

Purchase cost (or) Production cost: The cost of purchasing (producing) a unit of an item is known as purchase (production) cost.

Carrying (Holding) Cost: The carrying cost is associated with carrying (holding) inventory

Shortage (Stock out) Cost: The penalty cost for running out of stock is known as shortage cost

12 Mark Questions:

- A manufacturing company needs 2,500 units of particular component every year. The company 6 buys it at the rate of Rs. 30 per unit. The order processing cost for this part is estimates at Rs. 15 and the cost of carrying a part is in stock comes to about Rs. 4 per year. The company can manufacture this part internally. In that case, it saves 20% of the price of the product. However, it estimates a set up cost of Rs. 250 per production run. The annual production rate would be 4,800 units. However, the inventory holding costs remain unchanged. Determine (i) the EOQ and the optimal number of orders placed in a year (ii) the optimum production lot size and average duration of the production run (iii) Should the company manufactured the component internally or continue to purchase it from the supplier?
- The demand for an item in a company is 18,000 units per year, and the company can produce the items at a rate of 3,000 per month. The cost of one set-up is Rs. 500 and the holding cost of 1 unit per month is 15 paisa. The shortage cost of one unit is Rs. 20 per month. Determine (i) Optimum production batch quantity and the number of strategies (ii) Optimum cycle time and production time (iii) Maximum inventory level in the cycle and (iv) Total associated cost of the item is Rs. 20 per unit.
- Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity Unit costs (Rs) $0 \le Q_1 \le 800$ Re.1.00 $800 \le Q_2$ Re. 0.98

The yearly demand for the product is 1,600 units per year, cost of placing an order is Rs. 5, the cost

of storage is 10% per year.

4 Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit costs (Rs)
$0 \le Q_1 \le 500$	Rs.10.00
$500 \le Q_2$	Rs9.25

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The monthly demand for the product is 200 units, cost of placing an order is Rs. 350 the cost of storage is 2% of the unit cost.

5 Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit costs (Rs)
$0 \le Q_1 \le 200$	Rs.10.00
$200 \le Q_2 \le 750$	Rs 9.25
$750 \le Q_3$	Rs. 8.75

The monthly demand for the product is 200 units, cost of placing an order is Rs. 350 the cost of storage is 2% of the unit cost.

- A certain item costs Rs. 235/- per ton. The monthly requirements are 5 tons and each time the stock 6 is replenished, there is a set up cost of Rs. 1000/-. The cost of carrying inventory has been estimated at 10% of the average inventory per year. What is the optimum order quantity?
- 7 Find the optimum order quantity for a product for which the price breaks are as follows:

Quantity	Unit costs (Rs)
$0 \le Q_1 \le 100$	Rs.20.00
$100 \le Q_2 \le 200$	Rs 18.00
$200 \le Q_3$	Rs. 16.00

The monthly demand for the product is 400 units, cost of placing an order is Rs. 25/- per month, the cost of storage is 20% of the unit cost of the product

8 Two firms are competing for business under the condition that one firm's gain is another firm's loss. Firm A's pay off matrix is given below:

		Firm B			
		No Ad.	Medium Ad.	Heavy Ad.	
Firm A	No Advertising	10	5	-2	
	Medium Advertising	13	12	15	
	Heavy Advertising	16	14	10	

Suggest optimum strategy for the two firms and net outcome thereof using Dominance rule.

9 Solve the following game by linear programming techniques

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10 For the game with the following pay of matrix, determine the optimum strategies and the value of the game

	P.	2
P1	5	1
	3	4

In a small town there are only two stores, A and B that handles sundry goods. The total number of customers is equally divided between the two because price and quality of goods are equal. Both stores have good reputation in the community, and they render equally good customer service.

Assume that a gain of customer by A is a loss to B and vice versa. Both stores plan to run annul pre-diwali sales during the first week of November. Sales are advertised through a local newspaper, radio and television media. With the aid of an advertising firm store A constructed the game matrix given below.

	Strategy of B				
Strategy of A		Newspaper	Radio	Television	
	Newspaper	30	40	-80	
	Radio	0	15	-20	
	Television	90	20	50	

Determine the optimal strategies and the worth of such strategy for both A and B.

Obtain the optimal strategies for both-persons and the value of the game for Zero-sum two person game whose pay-off matrix is as follows:

1	-3
3	5
-1	6
4	1
2	2
-5	0

13 Solve the following 2 x 2 game

	Е	3
A	2	5
	7	3

14 Solve the following game using dominance property

A	В			
		Ι	II	II
	I	1	7	2
	Ii	6	2	7
	III	6	1	6

15 Consider a 'modified' form of 'matching biased coins' game problem. The matching player is paid Rs. 8 if the two coins turn both heads and Re. 1 if the coins turn both tails. The non-matching player is paid Rs. 3 when the two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy?

UNIT -V SEQUNCING PROBLEMS AND REPLACEMENT MODELS

2 Mark Questions:

- 1 Define sequencing problem.
- 2 Calculate the discounting factor over a period of one year if the money worth is 10 percent per year.
- 3 Define group replacement models.

Sometimes just after the complete breakdown of a system, the immediate replacement of the items may not be available, in such a circumstances a group replacement policy can be adopted

- 4 State the "no passing rule" in a sequencing problem.
 - It refers to the rule of maintain the order in which jobs are to be processed on a given machine.
- 5 Calculate the discount rate for an item that has to be replaced after nth year and the money is worth 5 percent per year.
- How should you deal with replacement of items that fail completely and suddenly? It is very difficult to predict that particular equipment will fail at a particular time. This uncertainty can be avoided by deriving the probability distribution of failure. Mortality tables are used to get life span of an equipment using probability distribution to minimize the total cost for replacement.
- 7 If the probability of failure of an item during 6 months period are 0.03, 0.07, 0.20, 0.40, 0.15 and 0.15. what is the expected life of an item?
- 8 **Define the terms:** a) **Number of machines b) Processing order c) Total elapsed time d) idle time Number of machine:** The number of machines refers to the number of service facilities through which a job must pass before it is assumed to be completed.

Processing order: it refers to the order in which machines are required for completing the job.

Total elapsed time: it is the time interval between starting the first job and completing the last job including the idle time in a particular sequence by the given set of machines.

Idle time: it is the time for which a machine does not have a job to process.

- 9 What are the assumptions in sequencing problem?
- 10 Give any two real situations that the replacement models are adopted.
 - Replacement of trucks, which are having frequent breakdowns.
 - Repalcement of electric light bulbs when they fail
 - Replacement of mechanized accounting system by computer system
- 11 Mention the replacement formula when the value of the money does not change with time.
- 12 What do you mean by present worth factor and payment compound factor?
- 13 Mention the replacement formula when the values of the money change with time.

12 Mark Questions:

1 Processing n Jobs through Two Machines:

In a factory, there are six jobs to perform, each of which should go through two machines A and B, in the order AB. The processing timings (in hours) for the jobs are given here. You are required to determine the sequence for performing the jobs that would minimize the total elapsed time. What is the total elapsed time?

Job	: J1	J2	J3	J4	J5	J6	
Machine A	:	1	3	8	5	6	3
Machine B	:	5	6	3	2	2	10

2 We have five jobs, each of which must go through the two machines A and B in the order AB.

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Processing times in hours are given in the table below:

Jobs	:	1	2	3	4	5
Machine A	:	5	1	9	3	10
Machine B	:	2	6	7	8	4

Determine a sequence for the five jobs that will minimize the elapsed time.

A book binder has one printing press, one binding machine and the manuscripts of a number of different books. The time required to performing the printing and binding operation for each book is shown below. Determine the order in which books should be processed, in order to minimize the total time required to turn out all the books

Book	:	1	2	3	4	5	6	
Printing Time (hrs)		: 30	120	50	20	90	100	
Binding time (hrs)		:	80	100	90	60	30	10

Processing n jobs through K machines:

Determine the optimal sequence of jobs that minimize the total elapsed time based on the following information processing time on machines is given in hours and passing not allowed:

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Job	: A	В	C	D	E	F	G
Machine	M1: 3	8	7	4	9	8	7
Machine	e M2: 4	3	2	5	1	4	3
Machine	e M3: 6	7	5	11	5	6	12

We have 4 jobs each of which has to go through the 6 machines in the order m1, m2, m3 M6. 12 processing time (In hours) is given below:

	m1	m2	m3	m4	m5	m6
job A	18	8	7	2	10	25
job B	17	6	9	6	8	19
job C	11	5	8	5	7	15
job D	20	4	3	4	8	12

Determine a sequence of these four jobs that minimize the total elapsed time

6 Solve the following sequencing problem when passing out is not allowed:

item	Machine A	Machine B	Machine C	Machine D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

7 Processing 2 Jobs through k machines:

Use the graphical method to minimize the time assed to process the following jobs on the machines shown (For each machine find the job which should be done first). Also calculate the total time elapsed to complete the jobs.

Job1	Sequence	A	В	C	D	E
3001	Time	3	4	2	6	2
Job	Sequence	В	C	A	D	Е
2	Time	5	4	3	2	6

8 Use the graphical method to minimize the time assed to process the following jobs on the machines

shown (For each machine find the job which should be done first). Also calculate the total time elapsed to complete the jobs.

Job1	Sequence	A	В	C	D	E
3001	Time	6	8	4	12	4
Job	Sequence	В	С	A	D	Е
2	Time	10	8	6	4	12

9	Case 1: Replacement	Policy when	Value of Money	does not c	hange with time:

A firm considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value, Rs.200. the running cost (maintenance and operating) in rupees are found from experience to be as follows:

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2 4 Year: 1 3 5 6 7 8 **Running Cost:** 200 500 800 1200 1800 2500 3200 4000

When should the machine be replace?

- 10 (a) Machine A costs Rs.9,000. Annual operating costs are Rs. 200 for the first year, and then 12 increases by Rs. 2000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine?
 - (b) Machine B costs Rs. 10000. Annual operating costs are Rs.400for the first year and then increases by Rs.800 every year. You now have a machine type A which is one year old. Should you replace it with B, if so when?

11 The Date collected in running a machine, the cost of which is Rs. 60,000, are given below.

Year:	1	2	3	4	5
Resale Value:	42000	30000	20400	14400	9650
Cost of Spares:	4000	4270	4880	5700	6800
Cost of labour:	14000	16000	18000	21000	250000

Determine the optimum period of replacement of the machine.

12 A fleet owner finds from past records that the costs per year of running a vehicle whose purchase 12 price is Rs. 50,000 are under:

Year 1 2 3 4 5 6 7 Running Costs (Rs) : 5000 6000 7000 9000 11500 16000 18000 Resale Value (Rs) 30000 15000 7500 9750 2000 2000 2000

Thereafter, running cost increases by Rs.2000, but resale value remains constant at Rs. 2000. At what age is the replacement due?

13 Case 2: Replacement Policy when Value of Money change with time:

A machine costs Rs 15000. The running costs for the different years are given below.

Year: 1 2 3 4 5 6 7 Running Cost: 2500 3000 4000 5000 6500 8000 10000

Find the optimum replacement period if the capital is worth 10% and no salvage value.

A manufacturer is offered two Machines A and B. A is priced at Rs.50000 and running costs are estimated at Rs.8000 for each of the first five years, increasing by Rs.2000 per year in the six and subsequent years. Machine B of the same capacity costs Rs.25000, but will have the running costs of Rs.12000 per year for six years increasing by Rs.2000 per year thereafter. If money is worth

10% per year, which machine should be purchased? (assume that the machine will eventually be sold for scrap at a negligible price)

15 The cost of new machine is Rs. 5,000. The maintenance cost of the nth year is given by $C_n = 5009n - 1$; n = 1,2,... suppose that the discount rate per year is 0.5. after how many years it will be economical to replace the machine by a new one?

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16 Case 3: Group replacement:

The following mortality rates have been observed for certain type of light bulbs

Week	1	2	3	4	5
% failing by the end of the week	10	25	50	80	100

There are 1000 bulbs in use and it costs Rs. 2 to replace an individual bulb which has burnt out. If all the bulbs were replaced simultaneously it would costs 50 paise per bulb. It is proposed to replace all bulbs at fixed intervals whether or not the have burnt out and to continue replacing burnt out bulbs as they fail. At what interval should all the bulbs be replaced?

17 A computer has a large number of electronic tubes. They are subject to mortality as given below:

Daniad	Age of failure	Probability of		
Period	(hours)	failure		
1	0-200	0.10		
2	200-400	0.26		
3	400-600	0.35		
4	600-800	0.22		
5	800-1000	0.07		

If the tubes are group replaced, the cost of replacement is Rs. 15 per tube. Group replacement can be done at fixed intervals in the night shift when the computer is not normally used. Replacement of individual tubes which fails in service costs Rs. 60 per tube. How frequently should the tube be replaced?